	Mean life	Standard deviation	Sample size
Brand A	2000 hrs	250 hrs	12
Brand B	2230 hrs	300 hrs	15

Do you think there is a significant difference in the quality of the two brands of bulbs?

[Delhi Univ., MBA, 1996]

10.33 Eight students were given a test in statistics, and after one month's coaching, they were given another test of the similar nature. The following table gives the increase in their marks in the second test over the first:

Roll No.	Increase in marks
1	2
2	–2
3	6
4	–8
5	12
6	5
7	–7
8	2

Do the marks indicate that the students have gained from the coaching?

10.34 An IQ test was administered to 5 persons before and after they were trained. The results are given below:

Candidate	I	II	III	IV	V
IQ before training :	110	120	123	132	125
IQ after training :	120	118	.25	136	121

Test whether there is any change in IQ level after the training programme. [Delhi Univ., MCom, 1998]

10.35 Eleven sales executive trainees are assigned selling jobs right after their recruitment. After a fortnight they are withdrawn from their field duties and given a month's training for executive sales. Sales executed by them in thousands of rupees before and after the training, in the same period are listed below:

Sales Before Training	Sales After Training
23	24
20	19
19	21
21	18
18	20
20	22
18	20
17	20
23	23
16	20
19	27

Do these data indicate that the training has contributed to their performance? [Delhi Univ., MCom, 1999]

Hints and Answers

10.23 Let $H_0 : \mu = 16$ and $H_1 : \mu \neq 16$ (Two-tailed test)
Given $n = 10$, $\bar{x} = 15.8$, $s = 0.50$. Using t-test

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{15.8 - 16}{0.50/\sqrt{10}} = -1.25$$

Since $t_{cal} = -1.25 >$ critical value $t_{\alpha/2} = -2.262$, at $df = 9$ and $\alpha/2 = 0.025$, the null hypothesis is accepted.

10.24 Let $H_0 : \mu = 27$ and $H_1 : \mu \neq 47$ (Two-tailed test)
Given $\bar{x} = 49$, $\Sigma(x - \bar{x})^2 = 52$, $n = 9$, and

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{(n-1)}} = \sqrt{\frac{52}{8}} = 2.55.$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{49 - 47}{2.55/\sqrt{9}} = 2.35$$

Since $t_{cal} = 2.35 >$ critical value $t_{\alpha/2} = 2.31$ at $\alpha/2 = 0.025$, $df = 8$, the null hypothesis is rejected.

10.25 Let $H_0 : \mu = 4{,}000$ and $H_1 : \mu \neq 4000$ (Two-tailed test)

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{(n-1)}} = \sqrt{\frac{3.12}{9}} = 0.589 \text{ and}$$

$$\bar{x} = \Sigma x/n = 44/10 = 4.4 \text{ (in Rs 000's)}.$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{4.4 - 4}{0.589/\sqrt{10}} = \frac{0.4}{0.186} = 2.150$$

Since $t_{cal} = 2.150 <$ critical value $t_{\alpha/2} = 2.62$ at $\alpha/2 = 0.025$ and $df = n - 1 = 9$, the null hypothesis is accepted.

10.26 Let $H_0 : \mu = 56$ and $H_1 : \mu \neq 46$ (Two-tailed test)
Given: $n = 16$, $\bar{x} = 53$ and $\Sigma(x - \bar{x})^2 = 135$. Thus

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{(n-1)}} = \sqrt{\frac{135}{15}} = 3$$

Applying t-test, $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{53 - 56}{3/\sqrt{16}} = -4$

Since $t_{cal} = -4 <$ critical value $t_{\alpha/2} = -2.13$ at $\alpha/2 = 0.025$, $df = 15$, the null hypothesis is rejected.

10.27 Let $H_0 : \mu = 5$ and $H_1 : \mu \neq 5$ (Two-tailed test)
Given $n = 10$, $\bar{x} = 5.02$ and $s = 0.002$.

Applying t-test, $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{5.02 - 5}{0.002/\sqrt{10}} = 33.33$

Since $t_{cal} = 33.33 >$ critical value $t_{\alpha/2} = 1.833$ at $\alpha/2 = 0.025$ and $df = 9$, the null hypothesis is rejected.

10.28 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)
Given $n_1 = 20$, $s_1 = 20$, $\bar{x}_1 = 170$; $n_2 = 18$, $s_2 = 25$, $\bar{x}_2 = 205$, Applying t-test,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s/\sqrt{n}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= \frac{170-205}{22.5}\sqrt{\frac{20\times18}{20+18}} = \frac{-35}{22.5}\sqrt{\frac{360}{38}} = -4.8$$

$$s = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{19\,(20)^2+17\,(25)^2}{20+18-2}} = \sqrt{\frac{18,225}{36}} = 22.5$$

Since $t_{cal} = -4.8 <$ critical value $t_{\alpha/2} = -1.9$ at $\alpha/2 = 0.025$ and $df = n_1 + n_2 - 2 = 36$, the null hypothesis is rejected.

10.29 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Given $n_1 = 9$, $\bar{x}_1 = 196.42$, $\Sigma(x_1 - \bar{x}_1)^2 = 26.94$ and $n_2 = 7$, $\bar{x}_2 = 198.82$ and $\Sigma(x_2 - \bar{x}_2)^2 = 18.73$

$$t = \frac{\bar{x}_1-\bar{x}_2}{s}\sqrt{\frac{n_1n_2}{n_1+n_2}} = \frac{196.42-198.82}{1.81}\sqrt{\frac{9\times7}{9+7}}$$

$$= -\frac{2.40}{1.81}\sqrt{\frac{63}{16}} = -2.63$$

$$s = \sqrt{\frac{\Sigma\,(x_1-\bar{x}_1)^2+(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{26.94+18.73}{9+7-2}} = 1.81$$

Since $t_{cal} = -2.63 <$ critical value $t_{\alpha/2} = -2.145$ at $\alpha/2 = 0.025$ and $df = 14$, the null hypothesis is rejected.

10.30 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Given $n_1 = 4$, $s_1^2 = 42$, $\bar{x}_1 = 52$, and $n_2 = 9$, $s_2^2 = 56$, $\bar{x}_2 = 42$.

$$t = \frac{\bar{x}_1-\bar{x}_2}{s}\sqrt{\frac{n_1n_2}{n_1+n_2}} = \frac{52-42}{7.224}\sqrt{\frac{4\times9}{4+9}}$$

$$= \frac{10}{7.224}\sqrt{\frac{36}{13}} = 2.303$$

$$s = \sqrt{\frac{(n_1-1)\,s_1^2+(n_2-1)\,s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{3\times42+8\times56}{4+9-2}} = \sqrt{\frac{574}{11}} = 7.224$$

Since $t_{cal} = 2.303 >$ critical value $t_{\alpha/2} = 2.20$ at $\alpha/2 = 0.025$ and $df = n_1 + n_2 - 2 = 11$, the null hypothesis is rejected.

10.31 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Given $n_1 = 12$, $s_1 = 100$, $\bar{x}_1 = 1380$ and $n_2 = 15$, $s_2 = 120$, $\bar{x}_2 = 1320$. Applying t-test,

$$s = \sqrt{\frac{(n_1-1)\,s_1^2+(n_2-1)\,s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{11\,(100)^2+14\,(120)^2}{12+15-2}} = 111.64$$

$$t = \frac{\bar{x}_1-\bar{x}_2}{s}\sqrt{\frac{n_1n_2}{n_1+n_2}} = \frac{1380-1320}{111.64}\sqrt{\frac{12\times15}{12+15}}$$

$$= \frac{60}{111.64}\sqrt{\frac{180}{27}} = 1.39$$

Since $t_{cal} = 1.39 <$ critical value $t_{\alpha/2} = 2.485$ at $\alpha/2 = 0.025$ and $df = n_1 + n_2 - 2 = 25$, the null hypothesis is accepted.

10.32 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Given $n_1 = 12$, $\bar{x}_1 = 2000$, $s_1 = 250$ and $n_2 = 15$, $\bar{x}_2 = 2230$, $s_2 = 300$. Applying t-test,

$$s = \sqrt{\frac{(n_1-1)\,s_1^2+(n_2-1)\,s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{11\,(250)^2+14\,(300)^2}{12+15-2}} = 279.11$$

$$t = \frac{\bar{x}_1-\bar{x}_2}{s}\sqrt{\frac{n_1n_2}{n_1+n_2}}$$

$$= \frac{2000-2230}{279.11}\sqrt{\frac{12\times15}{12+15}}$$

$$= -\frac{230}{279.11}\sqrt{\frac{180}{27}} = -2.126$$

Since $t_{cal} = -2.126 <$ critical value $t_{\alpha/2} = -1.708$ at $\alpha/2 = 0.025$ and $df = n_1 + n_2 - 2 = 25$, the null hypothesis is rejected.

10.33 Let H_0 : Students have not gained from coaching

Roll No	Increase in marks, d	d^2
1	4	16
2	–2	4
3	6	36
4	–8	64
5	12	144
6	5	25
7	–7	49
8	2	4
	12	342

$$\bar{d} = \frac{\Sigma d}{n} = \frac{12}{8} = 1.5;$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1}-\frac{(\Sigma d)^2}{n\,(n-1)}} = \sqrt{\frac{342}{7}-\frac{(12)^2}{8\times7}} = 6.8$$

$$t = \frac{\bar{d}}{s/\sqrt{n}} = \frac{1.5}{6.8/\sqrt{8}} = 0.624$$

Since $t_{cal} = 0.622 <$ critical value $t_{\alpha/2} = 1.895$ at $\alpha/2 = 0.025$ and $df = 7$, null hypothesis is accepted.

10.34 Let H_0 : No change in IQ level after training programme

IQ Level		d	d^2
Before	*After*		
110	120	10	100
120	118	−2	4
123	125	2	4
132	136	4	16
125	121	−4	16
		10	140

$$\bar{d} = \frac{\Sigma d}{n} = \frac{10}{5} = 2$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{140}{4} - \frac{(10)^2}{5 \times 4}} = 5.477$$

$$t = \frac{\bar{d}}{s/\sqrt{n}} = \frac{2}{5.477/\sqrt{5}} = 0.817$$

Since $t_{cal} = 0.814 <$ critical value $t_{\alpha/2} = 4.6$ at $\alpha/2 = 0.025$ and $df = 4$, the null hypothesis is accepted.

10.35 Let H_0 : Training did not improve the performance of the sales executives

Difference in Sales, d	d^2
1	1
−1	1
2	4
−3	9
2	4
2	4
2	4
3	9
0	0
4	16
8	64
= 20	= 116

$$\bar{d} = \frac{\Sigma d}{n} = \frac{20}{11} = 1.82$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{116}{10} - \frac{(20)^2}{11 \times 10}} = 2.82$$

$$t = \frac{\bar{d}}{s/\sqrt{n}} = \frac{1.82}{2.82/\sqrt{11}} = 2.14$$

Since $t_{cal} = 2.14 <$ critical value $t_{\alpha/2} = 2.23$ at $\alpha/2 = 0.025$ and $df = 10$, the null hypothesis is accepted.

10.11 HYPOTHESIS TESTING BASED ON F-DISTRIBUTION

In several statistical applications we might require to compare population variances. For instance, (i) variances in product quality resulting from two different production processes; (ii) variances in temperatures for two heating devices; (iii) variances in assembly times for two assembly methods, (iv) variance in the rate of return on investment of two types of stocks and so on, are few areas where comparison of variances is needed.

When independent random samples of size n_1 and n_2 are drawn from two normal populations, the ratio

$$F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$$

follow F-distribution with $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$ degrees of freedom, where s_1^2 and s_2^2 are two sample variances and are given by

$$s_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} \text{ and } s_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$$

F-test: A hypothesis test for comparing the variance of two independent populations with the help of variances of two small samples.

If two normal populations have equal variances, i.e. $\sigma_1^2 = \sigma_2^2$, then the ratio

$$F = \frac{s_1^2}{s_2^2}\,;\; s_1 > s_2$$

has a probability distribution in repeated samplling that is known as F-distribution with $n_1 - 1$ degrees of freedom for numerator and $n_2 - 1$ degrees of freedom for denominator. For computational purposes, a larger sample variance is placed in the numerator so that ratio is always equal to or more than one.

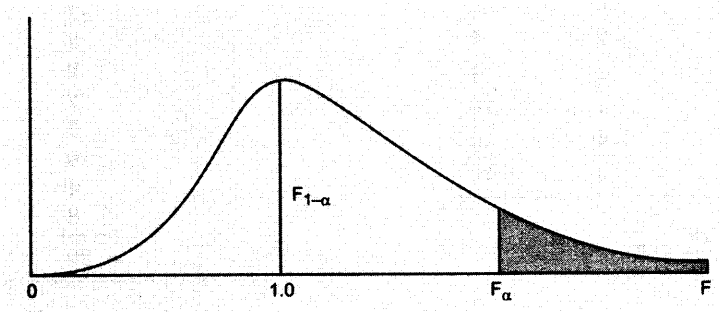
Assumptions: Few assumptions for the ratio s_1^2/s_2^2 to have an F-distribution are as follows:

(i) Independent random samples are drawn from each of two normal populations

(ii) The variability of the measurements in the two populations is same and can be measured by a common variance σ^2, i.e. $\sigma_1^2 = \sigma_2^2 = \sigma^2$

The F-distribution, also called *variance ratio distribution*, is not symmetric and the F values can never be negative. The shape of any F-distribution depends on the degrees of freedom of the numerator and denominator. A typical graph of a F-distribution is shown in Fig. 10.10 for equal degrees of freedom for both numerator and denominator.

Figure 10.10
F-distribution for *n* Degrees of Freedom



10.11.1 Properties of F-distribution

1. The total area or probability under the curve is unity. The value of F-test statistic denoted by F_α at a particular level of significance α, provides an area or probability of α to the right of the stated F_α value.
2. The F-distribution is positively skewed with a range 0 to ∞ and its degree of skewness decreases with the increase in degrees of freedom v_1 for numerator and v_2 for denominator. For $v_2 \geq 30$, F-distribution is approximately normal.
3. The sample variances s_1^2 and s_2^2 are the unbiased estimates of population variance. Since $s_1 > s_2$, the range of F-distribution curve is from 1 to ∞.
4. If the ratio s_1^2/s_2^2 is nearly equal to 1, then it indicates little evidence that σ_1^2 and σ_2^2 are unequal. On the other hand, a very large or very small value for s_1^2/s_2^2 provides evidence of difference in the population variances.
5. The F-distribution discovered by Sir Ronald Fisher is merely a transformation of the original Fisher's *z*-distribution and is written as

$$z = \frac{1}{2}\log_e F = \frac{1}{2}\log_e \frac{s_1^2}{s_2^2} = \frac{1}{2}\log_e\left(\frac{s_1}{s_2}\right)^2$$

$$= \log_e\left(\frac{s_1}{s_2}\right) = \log_{10}\left(\frac{s_1}{s_2}\right)\log_e 10 = 2.3026\log_{10}\left(\frac{s_1}{s_2}\right)$$

The probability density function of F-distribution is given by

$$f(\text{F}) \text{ or } y = y_0 \frac{e^{v_1 z}}{(v_1 e^{2Z} + v_2)^{-(v_1+v_2)/2}}, \quad -\infty < z < \infty$$

where y_0 is a constant depending on the values of degrees of freedom, $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$, such that the area under the curve is unity.

6. The mean and variances of the F-distribution are

$$\text{Mean } \mu = \frac{v_2}{v_2 - 2}, \; v_2 > 2$$

and

$$\text{Variance } \sigma^2 = \frac{2v_2^2(v_1 + v_2 - 2)}{v_1(v_2 - v_1)^2(v_2 - 4)}, \; v_2 > 4$$

This implies that F-distribution has no mean for $v_2 \leq 2$ and no variance for $v_2 \leq 4$.

7. The reciprocal property

$$F_{1-\alpha\,(v_2, v_1)} = \frac{1}{F_{\alpha\,(v_2, v_1)}}$$

of F-distribution helps to identify corresponding lower (left) tail F values from the given upper (right) tail F values. For example, if $\alpha = 0.05$, then for $v_1 = 24$ and $v_2 = 15$, then $F_{0.95(15,\,24)} = 2.11$. Thus $F_{0.95(24,\,15)} = 1/2.11 = 0.47$.

8. The variance of F with 1 and n degrees of freedom is distributed same as t-distribution with n degrees of freedom.

10.11.2 Comparing Two Population Variances

How large or small must the ratio s_1^2/s_2^2 be for sufficient evidence to exist to the null hypothesis is stated below:

Null hypothesis	Alternative hypothesis	
$H_0 : \sigma_1^2 = \sigma_2^2$	$H_1 : \sigma_1^2 > \sigma_2^2$ or $\sigma_1^2 < \sigma_2^2$	(One-tailed Test)
$H_0 : \sigma_1^2 = \sigma_2^2$	$H_1 : \sigma_1^2 \neq \sigma_2^2$	(Two-tailed Test)

To conduct the test, random samples of size n_1 and n_2 are drawn from population 1 and 2 respectively. The statistical test of the null hypothesis H_0, uses the test statistic $F = s_1^2/s_2^2$, where s_1^2 and s_2^2 are the respective sample variances.

Decision rules: The criteria of acceptance or rejection of null hypothesis H_0 are as under:

1. Accept H_0 if the calculated value of F-test statistic is less than its critical value $F_{\alpha(v_1, v_2)}$, i.e. $F_{cal} < F_\alpha$ for one-tailed test.

   The critical value of F_α is based on degrees of freedom of numerator $df_1 = n_1 - 1$ and degrees of freedom of denominator $df_2 = n_2 - 1$. These values can be obtained from F-Tables (See appendix).

   As mentioned earlier, the population with larger variance is considered as population 1 to ensure that a rejection of H_0 can occur only in the right (upper) tail of the F-distribution curve. Even though half of the rejection region (the area $\alpha/2$ to its left) will be in the lower tail of the distribution. It is never used because using the population with larger sample variance as population 1 always places the ratio s_1^2/s_2^2 in the right-tail direction.
2. $H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 > \sigma_2^2$ (One-tailed test)

   The null hypothesis is setup so that the rejection region is always in the upper tail of the distribution. This helps us in considering the population with larger variance in the alternative hypothesis.

Confidence Interval: An interval estimate of all possible values for a ratio σ_1^2/σ_2^2 of population variances, is given by

$$\frac{s_1^2/s_2^2}{F_{(1-\alpha)}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2/s_2^2}{F_\alpha}$$

where F values are based on a F-distribution with $(n_1 - 1)$ and $(n_2 - 1)$ degrees of freedom and $(1 - \alpha)$ confidence coefficient.

Example 10.29: A research was conducted to understand whether women have a greater variation in attitude on political issues than men. Two independent samples of 31 men and 41 women were used for the study. The sample variances so calculated were 120 for women and 80 for men. Test whether the difference in attitude toward political issues is significant at 5 per cent level of significance.

Solution: Let us take the hypothesis that the difference is not significant, that is,

$$H_0 : \sigma_w^2 = \sigma_m^2 \quad \text{and} \quad H_1 : \sigma_w^2 > \sigma_m \quad \text{(One-tailed test)}$$

The F-test statistic is given by $F = \dfrac{s_1^2}{s_2^2} = \dfrac{120}{80} = 1.50$

Since variance for women is in the numerator, the F-distribution with $df_1 = 41 - 1 = 40$ in the numerator and $df_2 = 31 - 1 = 30$ in the denominator will be used to conduct he one-tailed test.

The critical (table) value of $F_{\alpha = 0.05} = 1.79$ at $df_1 = 40$ and $df_2 = 30$. The calculated value of $F = 1.50$ is less than its critical value $F = 1.79$, the null hypothesis is accepted. Hence, the results of the research do not support the belief that women have a greater variation in attitude on political issues than men.

Example 10.30: The following figures relate to the number of units of an item produced per shift by two workers A and B for a number of days

A: 19 22 24 27 24 18 20 19 25

B: 26 37 40 35 30 30 40 26 30 35 45

Can it be inferred that worker A is more stable compared to worker B? Answer using the F-test at 5 per cent level of significance.

Solution: Let us take the hypothesis that the two workers are equally stable, that is,

$$H_0 : \sigma_A^2 = \sigma_B^2 \text{ and } H_1 : \sigma_A^2 \neq \sigma_B^2 \text{ (One-tailed test)}$$

The calculations for population variances σ_A^2 and σ_B^2 of the number of units produced by workers A and B, respectively are shown in Table 10.9.

Table 10.9: Calculation of σ_A^2 and σ_B^2

Worker A x_1	$x_1 - \bar{x}_1 = x_1 - 22$	$(x_1 - 22)^2$	Worker B x_2	$x_2 - \bar{x}_2 = x_2 - 34$	$(x_2 - \bar{x}_2)^2$
19	–3	9	26	–8	64
22	0	0	37	3	9
24	2	4	40	6	36
27	5	25	35	1	1
24	2	4	30	–4	16
18	–4	16	30	–4	16
20	–2	4	40	6	36
19	–3	9	26	–8	64
25	3	9	30	–4	16
			35	1	1
			45	11	121
198	0	80	374	0	380

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{198}{9} = 22; \qquad \bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{374}{11} = 34$$

$$s_A^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{80}{9-1} = 10; \qquad s_B^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{380}{11-1} = 38$$

Applying F-test statistic, we have

$$F = \frac{s_B^2}{s_A^2} = \frac{38}{10} = 3.8 \text{ (because } s_B^2 > s_A^2)$$

The critical value $F_{0.05(10, 8)} = 3.35$ at $\alpha = 5$ per cent level of significance and degrees of freedom $df_A = 8$, $df_B = 10$. Since the calculated value of F is more than its critical value, the null hypothesis is rejected. Hence we conclude that worker A is more stable than worker B, because $s_A^2 < s_B^2$.

Self-Practice Problems 10D

10.36 The mean diameter of a steel pipe produced by two processes, A and B, is practically the same but the standard deviations may differ. For a sample of 22 pipes produced by A, the standard deviation is 2.9 m, while for a sample of 16 pipes produced by B, the standard deviation is 3.8 m. Test whether the pipes produced by process A have the same variability as those of process B.

10.37 Tests for breaking strength were carried out on two lots of 5 and 9 steel wires respectively. The variance of one lot was 230 and that of the other was 492. Is there a significant difference in their variability?

10.38 Two random samples drawn from normal population are:

Sample 1	*Sample 2*
20	27
16	33
26	42
27	35
23	32
22	34
18	38
24	28
25	41
19	43
	30
	37

Obtain estimates of the variances of the population and test whether the two population have the same variance.

10.39 In a sample of 8 observations, the sum of the squared deviations of items from the mean was 94.50. In another sample of 10 observations the value was found to be 101.70. Test whether the difference is significant at 5 per cent level of significance (at 5 per cent level level of significance critical value of F for $v_1 = 3$ and $v_2 = 9$ degrees of freedom is 3.29 and for $v_1 = 8$ and $v_2 = 10$ degrees of freedom, its value is 3.07).

10.40 Most individuals are aware of the fact that the average annual repair costs for an automobile depends on the age of the automobile. A researcher is interested in finding out whether the variance of the annual repair costs also increases with the age of the automobile. A sample of 25 automobiles that are 4 years old showed a sample variance for annual repair cost of Rs 850 and a sample of 25 automobiles that are 2 years old showed a sample variance for annual repair costs of Rs 300. Test the hypothesis that the variance in annual repair costs is more for the older automobiles, for a 0.01 level of significance.

10.41 The standard deviation in the 12-month earnings per share for 10 companies in the software industry was 4.27 and the standard deviation in the 12-month earning per share for 7 companies in the telecom industry was 2.27. Conduct a test for equal variance at $\alpha = 0.05$. What is you conclusion about the variability in earning per share for two industries.

Hints and Answers

10.36 LetH_0 : There is no difference in the variability of diameters produced by process A and B, i.e.

$H_0 : \sigma_A^2 = \sigma_B^2$ and $H_1 : \sigma_A^2 \neq \sigma_B^2$

Given $\sigma_A = 2.9, n_1 = 22, df_A = 21;\ \sigma_B = 3.8,$
$n_2 = 16, df_B = 21.$

$$s_A^2 = \frac{n_1}{n_1 - 1}\sigma_A^2 = \frac{22}{22-1}(2.9)^2 = \frac{22}{21}(8.41) = 8.81$$

$$s_2^2 = \frac{n_2}{n_2 - 1}\sigma_B^2 = \frac{16}{16-1}(3.8)^2 = \frac{16}{15}(14.44) = 15.40$$

$$F = \frac{s_2^2}{s_1^2} = \frac{15.40}{8.81} = 1.75$$

Since the calculated value $f = 1.75$ is less than its critical value $F_{0.05\,(15,\,21)} = 2.18$, the null hypothesis is accepted.

10.37 Let H_0 : No significant variability in the breaking strength of wires

Given $n_1 = 5, \sigma_1^2 = 230, df_1 = 4; n_2 = 9, \sigma_2^2 = 492,$
$df_2 = 8$

$$F = \frac{\sigma_2^2}{\sigma_1^2} = \frac{492}{230} = 2.139$$

Since calculated value F = 2.139 is less its critical value $F_{0.05(8,\,4)} = 6.04$ the null hypothesis is accepted.

10.38 Let H_0 : Two populations have the same variance, i.e. $H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 \neq \sigma_2^2$.

$$\text{Sample 1: } \bar{x}_1 = \frac{\Sigma x_1}{10} = 22;$$

$$s_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{120}{9} = 13.33, df_1 = 9$$

$$\text{Sample 2: } x_2 = \frac{\Sigma x_1}{12} = 35;$$

$$s_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{314}{11} = 28.54, df_2 = 11$$

$$F = \frac{s_2^2}{s_1^2} = \frac{28.54}{13.33} = 2.14$$

Since calculated value F = 2.14 is less than its critical value $F_{0.05\,(11,\,9)} = 4.63$, the null hypothesis is accepted.

10.39 Let H_0 : The difference is not significant

Sample 1: $n_1 = 8, \Sigma(x_1 - \bar{x}_1)^2 = 94.50, v_1 = 7$

Sample 2: $n_1 = 10, \Sigma(x_2 - \bar{x}_2)^2 = 101.70, v_2 = 9$

$$\therefore\ s_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{94.50}{7} = 13.5;$$

$$s_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{101.70}{9} = 11.3$$

$$F = \frac{s_1^2}{s_2^2} = \frac{13.5}{11.3} = 1.195$$

Since the calculated value F = 1.195 is less than its critical value $F_{0.05\,(7,\,9)} = 3.29$, the null hypothesis is accepted.

10.40 Let H_0 : No significant difference in the variance of repair cost, $H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 > \sigma_2^2$

s_1^2 = Rs 850; s_2^2 = Rs 300

$n_1 = 25, df_1 = 24; n_2 = 25, df_2 = 24$

$$F = \frac{s_1^2}{s_2^2} = \frac{850}{300} = 2.833$$

Since the calculated value F = 2.833 is more than its critical value $F_{0.01(24,\,24)} = 2.66$, the null hypothesis is rejected.

10.41 Let H_0 : No significant difference of variability in earning per share for two industries,

$H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 \neq \sigma_2^2$

Software industry : $s_1^2 = (4.27)^2 = 18.23$, $n_1 = 10, df_1 = 9$

Telecom industry : $s_2^2 = (2.27)^2 = 5.15$, $n_2 = 7, df_2 = 6$

$$\therefore \quad F = \frac{s_1^2}{s_2^2} = \frac{18.23}{5.15} = 3.54$$

Since the calculated value F = 3.54 is less than its critical value $F_{0.05\,(9,\,6)} = 4.099$, the null hypothesis is accepted.

Formulae Used

1. **Hypothesis testing for population mean with large sample ($n > 30$)**
   (a) Test statistic about a population mean μ
   - σ assumed known, $z = \dfrac{|\bar{x} - \mu|}{\sigma/\sqrt{n}}$
   - σ is estimated by s, $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

   (b) Test statistic for the difference between means of two populations
   - Standard deviation of $\bar{x}_1 - \bar{x}_2$ when σ_1 and σ_2 are known

   $$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

   Test statistic $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$

   - Standard deviation of $\bar{x}_1 - \bar{x}_2$ when $\sigma_1^2 = \sigma_2^2$

   $$\sigma_{\bar{x}_1 - \bar{x}_2} = \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

   - Point estimator of $\sigma_{\bar{x}_1 - \bar{x}_2}$

   $$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

   - Interval estimation for single population mean

   $\bar{x} \pm z_{\alpha/2}\, \sigma_{\bar{x}}$; σ is known

   $\bar{x} \pm z_{\alpha/2}\, s_{\bar{x}}$; σ is unknown

   - Interval estimation for the difference of means of two populations

   $(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sigma_{\bar{x}_1 - \bar{x}_2}$; σ_1 and σ_2 are known

   $(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} s_{\bar{x}_1 - \bar{x}_2}$; σ_1 and σ_2 are unknown

2. **Hypothesis testing for population proportion for large sample ($n > 30$)**
   (a) Test statistic for population proportion p

   $$z = \frac{\bar{p} - p}{\sigma_{\bar{p}}};\ \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$$

   (b) Test statistic for the difference between the proportions of two populations
   - Standard deviation of $\bar{p}_1 - \bar{p}_2$

   $$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

   - Point estimator of $\sigma_{\bar{p}_1 - \bar{p}_2}$

   $$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

   - Interval estimation of the difference between the proportions of two populations

   $$(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\, s_{\bar{p}_1 - \bar{p}_2}$$

   where all $n_1p_1, n_1(1-p_1), n_2p_2$, and $n_2(1-p_2)$ are more than or equal to 5
   - Test statistic for hypothesis testing about the difference between proportions of two populations $z = \dfrac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{\sigma_{\bar{p}_1 - \bar{p}_2}}$
   - Pooled estimator of the population proportion

   $$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}$$

   - Point estimator of $\sigma_{\bar{p}_1 - \bar{p}_2}$

   $$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

3. **Hypothesis testing for population mean with small sample ($n \leq 30$)**
   - Test statistic when s is estimated by s

   $$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

   where $s = \sqrt{\dfrac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}} = \sqrt{\dfrac{\Sigma(x_1 - \bar{x}_1)^2 + (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$

   - Test statistic for difference between the means of two population proportions

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

where $s_{\bar{x}_1 - \bar{x}_2}$ is the point estimator of $\sigma_{\bar{p}_1 - \bar{p}_2}$ when $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

- Interval estimation of the difference between means of two populations $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\, s_{\bar{x}_1 - \bar{x}_2}$

4. Hypothesis testing for matched samples (small sample case)
   Test statistic for matched samples

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}; \quad s_d = \sqrt{\frac{\Sigma (d - \bar{d})^2}{n - 1}}; \ \bar{d} = \frac{\Sigma d}{n}$$

5. Hypothesis testing for two population variances

$$F = s_1^2/s_2^2; \quad s_1^2 > s_2^2$$

Chapter Concepts Quiz

True or False

1. A tentative assumption about a population parameter is called the null hypothesis. (T/F)
2. As a general guideline, a research hypothesis should be stated as the alternative hypothesis. (T/F)
3. The equality part of the expression (either $\geq$, $\leq$, or $=$) always appears in the null hypothesis. (T/F)
4. Type I error is the probability of accepting null hypothesis when it is true. (T/F)
5. Type II error is the probability of accepting null hypothesis when it is true. (T/F)
6. The probability of making a Type I error is referred to as the level of significance. (T/F)
7. The estimated standard deviation of sampling distribution of a statistic is called standard error. (T/F)
8. Type I error is more harmful than Type II error. (T/F)
9. For a given level of significance, we can reduce β by increasing the sample size. (T/F)
10. If the cost of Type I error is large, a small level of significance should be specified. (T/F)
11. For a given sample size n, an attempt to reduce the level of significance results in an increase in β. (T/F)
12. For a given level of significance, change in sample size changes the critical value. (T/F)
13. The t-test statistic is used when $n \leq 30$ and the population standard deviation is known. (T/F)
14. The value of the test statistic that defines the rejection region is called critical region for the test. (T/F)

Multiple Choice

15. Sampling distribution will be approximately normal if sample size is
    (a) large (b) sufficiently large
    (c) small (d) none of these
16. Critical region is a region of
    (a) rejection (b) acceptance
    (c) indecision (d) none of these
17. The probability of Type II error is
    (a) α (b) β
    (c) $1 - \alpha$ (d) $1 - \beta$
18. The term $1 - \beta$ is called
    (a) level of the test (b) power of the test
    (c) size of the test (d) none of these
19. The test statistic to test $\mu_1 = \mu_2$ for normal population is
    (a) F-test (b) z-test
    (c) t-test (d) none of these
20. The usual notation the standard error of the sampling distribution is
    (a) $\sigma/\sqrt{n}$ (b) σ/n
    (c) σ^2 (d) none of these
21. If Type I and Type II errors are fixed, then the power of a test increases with
    (a) the increase of sample size
    (b) not related to sample size
    (c) the decrease of sample size
    (d) none of these
22. The asymptotic distribution of t-statistic with n degrees of freedom is
    (a) F (b) normal
    (c) z (d) t
23. If we reject the null hypothesis, we might be making
    (a) a Type II error (b) a Type I error
    (c) a correct decision (d) either (a) or (b)
24. The large sample test for testing $p_1 = p_2$ for normal population is
    (a) z-test (b) t-test
    (c) F-test (d) none of these
25. For test of hypothesis $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 < \mu_2$, the critical region at $\alpha = 0.05$ and $n > 30$, is
    (a) $z \leq 1.96$ (b) $z > 1.96$
    (c) $z \leq 1.645$ (d) $z > 1.645$

26. For test of hypothesis $H_0 : \mu_1 \le \mu_2$ and $H_1 : \mu_1 > \mu_2$, the critical region at $\alpha = 0.10$ and $n > 30$, is
(a) $z \le 1.96$ (b) $z > 1.96$
(c) $z \le -1.645$ (d) $z > 1.645$

27. The test statistic $F = s_1^2/s_2^2$ is used for testing the null hypothesis
(a) $H_0 : \mu_1 = \mu_2$ (b) $H_0 : \sigma_1^2 = \sigma_2^2$
(c) $H_0 : \sigma_1 = \sigma_2$ (d) none of these

28. Interval estimation of the difference between the proportion of two populations is given by
(a) $(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\, s_{\bar{p}_1 - \bar{p}_2}$ (b) $(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha}\, s_{\bar{p}_1 - \bar{p}_2}$
(c) $(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\, \sigma_{\bar{p}_1 - \bar{p}_2}$ (d) $(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha}\, \sigma_{\bar{p}_1 - \bar{p}_2}$

29. Interval estimation for a single population mean when σ is known is given by
(a) $\bar{x} \pm z_{\alpha}\, \sigma_{\bar{x}}$ (b) $\bar{x} \pm z_{\alpha/2}\, \sigma_{\bar{x}}$
(c) $\bar{x} \pm z_{\alpha}\, s_{\bar{x}}$ (d) $\bar{x} \pm z_{\alpha/2}\, s_{\bar{x}}$

30. Suppose for a particular hypothesis $\alpha = 0.05$ and $\beta = 0.10$. Then the power of this hypothesis test is
(a) 0.85 (b) 0.90
(c) 0.95 (d) none of these

31. If 2.81 is the critical value of test-statistic z, then the significance level of the test is
(a) 0.05 (b) 0.01
(c) 0.005 (d) none of these

32. To be assumed that a hypothesis test is working correctly, it is desired that the value of $1 - \beta$ should be close to
(a) 0.25 (b) .050
(c) .075 (d) 1.0

33. When a null hypothesis is $H_0 : \mu = \mu_0$, the alternative hypothesis can be
(a) $H_1: \mu \ge \mu_0$ (b) $H_1: \mu < \mu_0$
(c) $H_1: \mu \ne \mu_0$ (d) $H_1: \mu = \mu_0$

34. When using the sample proportion $\bar{p}$ to test the hypothesis, the standard error of $\bar{p}$ is:
(a) $\bar{p}\bar{q}/\sqrt{n}$ (b) $pq/\sqrt{n}$
(c) $\sqrt{\bar{p}\bar{q}/n}$ (d) $\sqrt{pq/n}$

Concepts Quiz Answers

1. T	**2.** T	**3.** T	**4.** F	**5.** T	**6.** F	**7.** T	**8.** T	**9.** F
10. T	**11.** T	**12.** T	**13.** T	**14.** F	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (b)
19. (c)	**20.** (a)	**21.** (a)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (b)
28. (a)	**29.** (b)	**30.** (b)	**31.** (c)	**32.** (d)	**33.** (b)	**34.** (c)		

Review-Self Practice Problems

10.42 A sample of size 25 yielded a mean equal to 33 and an estimated variance equal to 100. At the $\alpha = 0.01$ would we have reasons to doubt the claim that the population mean is not greater than 27?
[Kurukshetra Univ., MCom, 1996]

10.43 An educator claims that the average IQ of American college students is at most 110, and that in a study made to test this claim 150 American college students selected at random had an average IQ of 111.2 with a standard deviation of 7.2. Use a level of significance of 0.01 to test the claim of the educator.
[Delhi Univ., BA (H.Eco.), 1996]

10.44 Prices of shares (in Rs) of a company on different days in a month were found to be: 66, 65, 69, 70, 69, 71, 70, 63, 64, and 68. Test whether the mean price of the shares in the month is 65. *[Delhi Univ., MBA, 1999]*

10.45 500 apples are taken at random from a large basket and 50 are found to be bad. Estimate the proportion of bad apples in the basket and assign limits within which the percentage most probably lies.

10.46 The election returns showed that a certain candidate received 46 per cent of the votes. Determine the probability that a poll of (a) 200 and (b) 1000 people selected at random from the voting population would have shown a majority of votes in favour of the candidate.

10.47 A simple random sample of size 100 has mean 15, the population variance being 25. Find an interval estimate of the population mean with a confidence level of (i) 99 per cent, and (ii) 95 per cent. If population variance is not given, what should be known to find out the required interval estimates? *[CA Nov., 1988]*

10.48 A machine produced 20 defective articles in a batch of 400. After overhauling, it produced 10 defectives in a batch of 300. Has the machine improved?
[Madras Univ., MCom, 1996]

10.49 You are working as a purchase manager for a company. The following information has been supplied to you by two manufacturers of electric bulbs:

		Company A	*Company B*
Mean life (in hours)	:	1300	1288
Standard deviation (in hours)	:	82	93
Sample size	:	100	100

Which brand of bulbs you will prefer to purchase if your desire is to take a risk of 5 per cent.
[Delhi Univ., MCom, 1994; MBA, 1998, 2002]

10.50 The intelligence test given to two groups of boys and girls gave the following information:

	Mean Score	*Standard Deviation*	*Number*
Girls	75	10	50
Boys	70	12	100

Is the difference in the mean scores of boys and girls statistically significant? [*Delhi Univ., MBA, 1997*]

10.51 Two samples of 100 electric bulbs each has a mean length of life 1500 and 1550 hours and standard deviation of 50 and 60 hours. Can it be concluded that two brands differ significantly at 1 per cent level of significance in equality? [*Kurukshetra Univ., MBA, 1999*]

10.52 Two types of drugs were used on 5 and 7 patients for reducing their weight. Drug A was imported and drug B indigeneous. The decrease in the weights after using the drugs for six months was as follows:

Drug A :	10	12	13	11	14	10	9
Drug B :	8	9	12	14	15		

Is there a significant difference in the efficacy of the two drugs? If not, which drug should you buy? [*Osmania Univ., MBA, 1996*]

10.53 Samples of two different types of bulbs were tested for length of life, and the following data were obtained:

		Type I	*Type II*
Sample size	:	8	7
Sample mean	:	1234 hrs	1136 hrs
Sample S.D.	:	36 hrs	40 hrs

Is the difference in mean life of two types of bulbs significant? [*Delhi Univ., MBA, 1996*]

10.54 A college conducting both day and evening classes intends them to be identical. A sample of 100 day students yields examination results as: $\bar{x}_1 = 72.4$, $\sigma_1 = 14.8$. Similarly, a sample of 200 evening students yields examination results as: $\bar{x}_2 = 73.9, \sigma_2 = 17.9$. Are the day and evening classes statistically equal at $\alpha = 0.01$ level of significance?[*Sukhadia Univ., M.Com, 1989*]

10.55 A strength test carried out on samples of two yarns spun to the same count gave the following results:

	Sample Size	*Sample Mean*	*Sample Variance*
Yarn A	4	52	42
Yarn B	9	42	56

The strengths are expressed in pounds. Is the difference in mean strengths significant of the sources from which the samples are drawn? [*Delhi Univ., MBA, 1998*]

10.56 An investigation of the relative merits of two kinds to flashlight batteries showed that a random sample of 100 batteries of brand X lasted on the average 36.5 hours with a standard deviation of 1.8 hours, while a random sample of 80 batteries of brand Y lasted on the average 36.8 hours with a standard deviation of 1.5 hours. Use a level of significance of 0.05 to test whether the observed difference between the average lifetimes is significant. [*Delhi Univ, BA (H. Econ.), 1996*]

10.57 A company is interested in finding out if there is any difference in the average salary received by the managers of two divisions. Accordingly, samples of 12 managers of the first division and 10 managers of the second division where selected at random. The results are given below:

	First Division	*Second Division*
Sample size	12	10
Average monthly salary	12,500	11,200
Standard deviation	320	480

Apply the *t*-test to find out whether there is a significant difference in the average salary. [*Kumaon Univ., MBA, 1999*]

10.58 A random sample of 100 mill workers in Kanpur showed their mean wage to be Rs 3500 with a standard deviation of Rs 280. Another random sample of 150 mill workers in Mumbai showed the mean wage to be Rs 3900 with a standard deviation of Rs 400. Do the mean wages of workers in Mumbai and Kanpur differ significantly ? Use $\alpha = 0.05$ level of significance. [*Delhi Univ., MCom, 1999*]

10.59 The sales data of an item in six shops before and after a special promotional campaign are as under:

Shops	:	A	B	C	D	E	F
Before campaign	:	53	28	31	48	50	42
After campaign	:	58	29	30	55	56	45

Can the campaign be judged to be a success? Test at 5 per cent level of significance. [*Jammu Univ., MCom, 1997*]

10.60 As a controller of budget you are presented with the following data for budget variances (in Rs 000's)

Department	*Budgeted sales*	*Actual sales*
A	1000	900
B	850	880
C	720	650
D	1060	860
E	750	820
F	900	1000
G	620	700
H	600	540
I	700	690
J	700	730
K	950	850
L	1100	1080

Is there any reason that achievements against budgets are slipping? Take $\alpha = 0.05$ level of significance.

10.61 A certain medicine given to each of 12 patients resulted in the following increase in blood pressure: 2, 5, 8, –1, 3, 0, –2, 1, 5, 0, 4, 6. Can it be concluded that the medicine will, in general, be accompanied by an increase in blood pressure?

10.62 In a survey of buying habits, 400 women shoppers are chosen at random in a shopping mall A. Their average monthly food expenditure is Rs. 2500 with a standard deviation of Rs 400, for another group of 400 women shoppers chosen at random in shopping mall B, located in another area of the same city, the average monthly food expenditure is Rs 2200 with a standard deviation of Rs 550. Do the data present sufficient evidence to indicate that the average monthly food expenditure of the two populations of women shoppers are equal. Test at the 1 per cent level of significance.

10.63 The average monthly earnings for a women in managerial and professional positions is Rs 16,700. Do men in the same positions have average monthly earnings that are higher than those for women? A random sample of $n = 40$ men in managerial and professional positions showed $\bar{x}$ = Rs 17,250 and s = Rs 2346. Test the appropriate hypothesis using $\alpha = 0.01$.

10.64 The director (finance) of a company collected the following sample information to compare the daily travel expenses for the sales staff and the audit staff:

Sales (Rs) :	262	270	292	230	272	284	
Audit (Rs):	260	204	258	286	298	240	278

At the $\alpha = 0.10$ significance level, can he conclude that the mean daily expenses are greater for the sales staff.

10.65 The owener of the weight lifting centre claims that by taking a special vitamin, a weight lifter can increase his strength. Ten student athletes are randomly selected and given a test of strength using the standard bench press. After two-weeks of regular training, supplemented with the vitamin, they are tested again. The results are shown below:

Student:	1	2	3	4	5	6	7	8	9	10
Before :	190	250	345	210	114	126	186	116	196	125
After :	196	240	345	212	113	129	189	115	194	124

At the $\alpha = 0.01$ significance level, can we conclude that the special vitamin increased the strengh of the student athletes?

10.66 The manufacturer of the motorcycle advertises that the motercycle will average 87 kms per litre on long trips. The kms on eight long trips were 88, 82, 81, 87, 80, 78, 79 and 89. At the $\alpha = 0.05$ significance level, is the mean kilometer less than the advertised 87 kilometers per litre.

10.67 The variability in the amount of impurities present in a batch of chemical used for a particular process depends on the length of time the process is in operation. A manufacturer using two production lines 1 and 2 has made a slight adjustment to line 2, hoping to reduce the variability as well as the average amount of impurities in the chemical. Samples of $n_1 = 25$ and $n_2 = 25$ measurements from two batches yield following means and variances: $\bar{x}_1 = 3.2$, $s_1^2 = 1.04$ and $\bar{x}_2 = 3.0$, $s_2^2 = 0.51$. Do the data present sufficient evidence to indicate that the process variability is less for line 2?

10.68 A media research group conducted a study of the radio listening habits of men and women. It was discovered that the mean listening time for men is 35 minutes perday with a standard deviation of 10 minutes in a sample of 10 men studied. The mean listening time for 12 women studied was also 35 minutes with a standard deviation of 12 minutes. Can it be concluded that there is a difference in the variation in the number of minutes men and women listen to the radio at $\alpha = 0.10$ significance level?

Hints and Answers

10.42 Let H_0 : No significant difference between the and hypothesized population means

Given, $\bar{x} = 33, s = \sqrt{100} = 10, n = 25$, and $\mu = 27$.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{33 - 27}{10/\sqrt{25}} = 3$$

Since $t_{cal} = 3$ is more than its critical value t = 2.064 at $\alpha/2 = 0.025$ and $df = 24$, the H_0 is rejected

10.43 Let H_0 : No significant difference between the claim of the educator and the sample results

Given, $n = 150$, $\bar{x} = 111.2, s = 7.2$ and $\mu = 110$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{111.2 - 110}{7.2/\sqrt{150}} = 2.04$$

Since $z_{cal} = 2.04$ is less than its critical value $z_\alpha = 2.58$ at $\alpha = 0.01$, the H_0 is accepted.

10.44 Let $H_0 : \mu = 65$ and $H_1 : \mu \neq 65$

x	$d = x - 65$	d^2
66	1	1
65	0	0
69	4	16
70	5	25
69	4	16
71	6	36
70	6	36
63	–2	4
64	–1	1
68	3	9
	25	133

$$\bar{x} = A + \frac{\Sigma d}{n} = 65 + \frac{25}{10} = 67.5\ ;$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{133}{9} - \frac{(25)^2}{10 \times 9}} = 2.798$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{67.5 - 65}{2.798/\sqrt{10}} = 2.81$$

Since t_{cal} = 2.81 is more than its critical value $t_{\alpha/2}$ = 1.833 at $\alpha/2$ = 0.025 and df = 9, the H_0 is rejected.

10.45 Population of bad apples in the given sample, $p = 50/500 = 0.1; q = 0.9$

Standard error, $\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.1 \times 0.9}{500}} = 0.013$

Confidence limits: $p \pm 3\,\sigma_p = 0.1 \pm 3(0.013)$;

or $0.081 \le p \le 0.139$

10.46 (a) Let $H_0 : p = 0.46$ and $H_1 : p \neq 0.46$

Given $p = 0.46, q = 0.54, n = 200$;

$$\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.46 \times 0.54}{200}} = 0.0352$$

Since 101 or more indicates a majority, as a continuous variable let us consider it as 100.5 and therefore the proportion is 100.5/200 = 0.5025

$$P(x \ge 101) = P\left[z \ge \frac{\bar{p} - p}{\sigma_p}\right] = P\left[z \ge \frac{0.5025 - 0.46}{0.0352}\right]$$

$$= P[z \ge 1.21]$$

Required probability = 0.5000 – 0.3869 = 0.1131

(b) $\sigma_p = \sqrt{\frac{0.46 \times 0.54}{1000}} = 0.0158$

$$P(x \ge 1001) = P\left[z \ge \frac{\bar{p} - p}{\sigma_p}\right] = P\left[z \ge \frac{0.5025 - 0.46}{0.0158}\right]$$

$$= P[z \ge 2.69]$$

Required probability = 0.500 – 0.4964 = 0.0036

10.47 Given $n = 100$, $\bar{x} = 15$, $\sigma_2 = 25$;

Standard error $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 5/\sqrt{100} = 0.5$

99 per cent confidence limits: $\bar{x} \pm 2.58\,\sigma_{\bar{x}}$

15 ± 2.58 (0.5); 3.71 to 16.29

95 per cent Confidence limits: $\bar{x} \pm 1.96\,\sigma_{\bar{x}}$

= 15 ± 1.96(0.5); 14.02 to 15.98

10.48 Let H_0 : The machine has not improved after overhauling, $H_0 : p_1 = p_2$

Given, $p_1 = 20/400 = 0.050$, $p_2 = 10/300 = 0.033$

$$\therefore \quad p = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2} = \frac{20 + 10}{400 + 300} = 0.043;$$

$$q = 1 - p = 0.957$$

$$z = \frac{p_1 - p_2}{\sigma_{p_1 - p_2}} = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.050 - 0.033}{\sqrt{0.043 \times 0.957\left(\frac{1}{400} + \frac{1}{300}\right)}}$$

$$= \frac{0.050 - 0.033}{0.0155} = 1.096$$

Since z_{cal} = 1.096 is less than its critical value z_α = 1.96 at α = 5 per cent, the H_0 is accepted.

10.49 Let H_0 : No significant difference in the quality of two brands of bulbs, i.e. $H_0 : \mu_1 = \mu_2$

Given $n_1 = 100, s_1 = 82, \bar{x}_1 = 1300$ and $n_2 = 100$, $s_2 = 93, \bar{x}_2 = 1288$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1300 - 1288}{\sqrt{\frac{(82)^2}{100} + \frac{(93)^2}{100}}}$$

$$= \frac{12}{\sqrt{67.24 + 86.49}} = 0.968$$

Since z_{cal} = 0.968 is less than its critical value z_α = 1.96 at α = 5 per cent, the H_0 is accepted.

10.50 Let H_0 : The difference in the mean score of boys and girls is not significant,

Given $n_1 = 50, s_1 = 10, \bar{x}_1 = 75$ and $n_2 = 100, s_2 = 12$, $\bar{x}_2 = 70$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{75 - 70}{\sqrt{\frac{(10)^2}{50} + \frac{(12)^2}{100}}}$$

$$= \frac{5}{\sqrt{3.44}} = 2.695$$

Since z_{cal} = 2.695 is more than its critical value z_α = 2.58 at α = 5 per cent, the H_0 is rejected.

10.51 Let H_0 : There is no significant difference in the mean life of the two makes of bulbs,

Given $n_1 = 100, \bar{x}_1 = 1500, \sigma_1 = 50$ and $n_2 = 100$, $\bar{x}_2 = 1550, \sigma_2 = 60$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{1500 - 1550}{\sqrt{\frac{(50)^2}{100} + \frac{(60)^2}{100}}} = -\frac{50}{7.81} = -6.40$$

Since z_{cal} = – 6.40 is less than its critical value z_α = – 2.58 at α = 1 per cent, the H_0 is rejected.

10.52 Let H_0 : No significant difference in the efficacy of two drugs, that is, $H_0 : \mu_1 = \mu_2$

Drug A: $\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{60}{5} = 12$; $\Sigma(x_1 - \bar{x}_1)^2 = 10$

Drug B: $\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{77}{7} = 11$; $\Sigma(x_2 - \bar{x}_2)^2 = 44$

$$\therefore \quad s = \sqrt{\frac{\Sigma (x_1 - \bar{x}_1)^2 + (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}} = \sqrt{\frac{10 + 44}{5 + 7 - 2}}$$

$$= 2.324$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s}\sqrt{\frac{n_1 n_2}{n_1 + n_2}} = \frac{12 - 11}{2.324}\sqrt{\frac{5 \times 7}{5 + 7}}$$

$$= \frac{1}{2.324}\sqrt{\frac{35}{12}} = 0.735$$

Since $t_{cal} = 0.735$ is less than its critical value $t = 2.228$ at $\alpha = 5$ per cent and $df = n_1 + n_2 - 2 = 10$, the H_0 is accepted.

10.53 Let H_0 : No significant difference in the mean life of the two types of bulbs, i.e. $H_0 : \mu_1 = \mu_2$

Type I: $n_1 = 8$, $\bar{x}_1 = 1234$, $s_1 = 36$; Type II: $n_2 = 7$, $\bar{x}_2 = 1136$, $s_2 = 40$

$$\therefore \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{7 \times 36 + 6 \times 40}{8+7-2}} = 37.9$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s}\sqrt{\frac{n_1 n_2}{n_1+n_2}} = \frac{1234-1136}{37.9}\sqrt{\frac{8\times 7}{8+7}}$$

$$= \frac{98}{37.9} \times 1.932 = 5$$

Since $t_{cal} = 5$ is more than its critical value $t_\alpha = 2.16$ at $\alpha = 5$ per cent and $df = n_1 + n_2 - 2 = 13$, the H_0 is rejected.

10.54 Let H_0 : Two means are statistically equal, that is, $H_0 : \mu_1 = \mu_2$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{72.4 - 73.9}{\sqrt{\frac{(14.8)^2}{100} + \frac{(17.9)^2}{200}}} = -0.77$$

Since $z_{cal} = -0.77$ is more than its critical value $z_\alpha = -2.58$ at $\alpha = 1$ per cent, the H_0 is accepted.

10.55 Let H_0 : The difference in the mean strength of the two yarns is not significant,

Yarn A : $n_1 = 4$, $\bar{x}_1 = 52$, $s_1^2 = 42$; Yarn B: $n_2 = 9$, $\bar{x}_2 = 42$, $s_2^2 = 56$

$$\therefore \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{3\times 42 + 8 \times 56}{4+9-2}} = 7.22$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s} \times \sqrt{\frac{n_1 n_2}{n_1+n_2}} = \frac{52-42}{7.22}\sqrt{\frac{4\times 9}{4+9}}$$

$$= \frac{10}{7.22}\sqrt{\frac{36}{13}} = 2.3$$

Since $t_{cal} = 2.3$ is more than its critical value $t_\alpha = 1.796$ at $\alpha = 5$ per cent and $df = 11$, the H_0 is rejected.

10.56 Let H_0 : The difference in the average life of two makes of batteries is not significant,

Given $n_1 = 100$, $\bar{x}_1 = 36.5$, $\sigma_1 = 1.8$ and $n_2 = 80$, $\bar{x}_2 = 36.81$, $\sigma_2 = 1.5$

$$\therefore \quad z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{36.5 - 36.81}{\sqrt{\frac{(1.8)^2}{100} + \frac{(1.5)^2}{80}}} = \frac{1.31}{0.246} = 1.22$$

Since $z_{cal} = 1.22$ is less than its critical value $z = 1.96$ at $\alpha = 5$ per cent, the H_0 is accepted.

10.57 Let H_0 : The difference in the average salary of the two divisions is not significant,

Given $n_1 = 12$, $\bar{x}_1 = 12{,}500$, $s_1 = 320$ and $n_2 = 10$, $\bar{x}_2 = 11{,}200$, $s_2 = 480$

$$\therefore \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{11\times(320)^2 + 9 \times (480)^2}{12+10-2}} = 400$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s}\sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$= \frac{12{,}500 - 11{,}200}{400}\sqrt{\frac{12\times 10}{12+10}} = 7.59$$

Since $t_{cal} = 7.59$ is more than its critical value $t_\alpha = 2.228$ at $\alpha = 5$ per cent and $df = n_1 + n_2 - 2 = 20$, the H_0 is rejected.

10.58 Let H_0 : Overhauling has not improved the performance of the machine.

Given $p_1 = 10/200 = 0.05$; $p_2 = 4/100 = 0.04$;

$$\therefore \quad p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{10+4}{200+100} = 0.047;$$

$$q = 1 - p = 0.953$$

$$\sigma_p = \sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{0.047 \times 0.953\left(\frac{1}{200} + \frac{1}{100}\right)} = 0.027$$

$$z = \frac{p_1 - p_2}{\sigma_p} = \frac{0.05 - 0.04}{0.027} = 0.370$$

Since $z_{cal} = 0.370$ is less than its critical value $z = 1.96$ at $\alpha = 5$ per cent, the H_0 is accepted.

10.59 Let H_0 : The promotional campaign has not been success, i.e. $\mu_1 = \mu_2$

d	:	5	1	−1	7	6	3 = 21
d^2	:	25	1	1	49	36	9 = 121

$$\bar{d} = \frac{\Sigma d}{n} = \frac{21}{6} = 3.5;$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{121}{5} - \frac{(21)^2}{5\times 6}} = 3.08$$

$$t = \frac{\bar{d}}{s/\sqrt{n}} = \frac{3.5}{3.08/\sqrt{6}} = 2.78$$

Since t_{cal} = 2.78 is more than its critical value t = 2.571 at α = 5 per cent and $df = n - 1 = 5$, the H_0 is rejected. Campaign is successful.

10.60 Let H_0 : Achievements in sales against budgets are slipping, i.e., $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 < \mu_2$

Department	*d*	d^2
A	-100	10,000
B	30	900
C	-70	4900
D	-200	40,000
E	70	4900
F	100	10,000
G	80	6400
H	-60	3600
I	-10	100
J	30	900
K	-100	10,000
L	-20	400
	-250	92,100

$$d = \frac{\Sigma d}{n} = \frac{250}{12} = 20.83;$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{92{,}100}{11} - \frac{(-250)^2}{12 \times 11}} = 88.87$$

$$\therefore \quad t = \frac{\bar{d}}{s/\sqrt{n}} = \frac{20.83}{88.87/\sqrt{12}} = 0.812$$

Since t_{cal} = 0.812 is less than its critical value t = 1.79 at α = 5 per cent and df = 11, the H_0 is accepted.

10.61 Let H_0 : Medicine is not accompanied by an increase in blood pressure, i.e.

$H_0 : \mu_1 = \mu_2$ and $H_0 : \mu_1 < \mu_2$

d	d^2
2	4
5	25
8	64
-1	1
3	9
0	0
-2	4
1	1
5	25
0	0
4	16
6	36
31	185

$$\bar{d} = \frac{\Sigma d}{n} = \frac{31}{12} = 2.58;$$

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{185}{11} - \frac{(31)^2}{12 \times 11}} = 3.08$$

$$\therefore \quad t = \frac{\bar{d}}{s/\sqrt{n}} = \frac{2.58}{3.08/\sqrt{12}} = 2.89$$

Since t_{cal} = 2.89 is more than its critical value t = 1.796 at α = 5 per cent and df = 11, the H_0 is rejected. Medicine increases blood pressure.

10.62 H_0: $\mu_1 = \mu_2$ and H_0: $\mu_1 \neq \mu_2$; μ_1 and μ_2 = average monthly food expenditures of population 1 and 2 respectively.

Given: $n_1 = 400$, $\bar{x}_1 = 2500$, $s_1 = 400$ and $n_2 = 400$, $\bar{x}_2 = 2200$, $s_2 = 550$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_2^1}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2500 - 2200}{\sqrt{\frac{(400)^2}{400} + \frac{(550)^2}{400}}}$$

$$= \frac{300 \times 20}{\sqrt{462500}} = \frac{6000}{680.07} = 8.822$$

Since z_{Cal} (= 8.822) > $z_{\alpha/2}$ (= 2.58) at α = 0.01, H_0 is rejected. Hence, average monthly expenditure of two population of women shopper differ significantly.

10.63 H_0: μ = 16,700 and H_1: μ > 16,700; μ = average monthly salary for men

Given n = 400, $\bar{x}$ = 17,250 and s = 2346.

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{17{,}250 - 16{,}700}{2346/\sqrt{400}}$$

$$= \frac{550}{117.3} = 4.68$$

Since z_{cal} (=4.68) > z_α = 2.58, H_0 is rejected and hence we conclude that the average monthly earnings for men are significantly higher than for women.

10.64 H_0: $\mu_s \leq \mu_a$ and H_1: $\mu_s > \mu_a$; $df = 6 + 7 - 1 = 11$

$$s = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(6-1)(12.2)^2 + (7-1)(15.8)^2}{6 + 7 - 2} = 203.82$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{142.5 - 130.3}{\sqrt{203.82\left(\frac{1}{6} + \frac{1}{7}\right)}}$$

$$= 1.536$$

Since t_{cal} (= 1.536) > t_a (= 1.363) at df = 11, reject H_0. The men daily expenses are greater for sales staff.

10.65 H_0: $\mu_a \leq 0$ and H_0: $\mu_a > 0$. Calculate $\bar{d}$ = 0.10 and s_d = 4.28. Apply test statistic

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{0.10}{4.28/\sqrt{10}} = 0.07.$$

Since t_{cal} (= 0.07) < t_α (= 2.821) at df = 9, reject H_0. Hence there has been no increase.

10.66 H_0: $\mu \geq 87$ and H_1: $\mu < 87$

Calculate $\bar{x} = \sum d/n = 664/8 = 83.0$,

$$s = \sqrt{\frac{\Sigma d^2}{n-1} - \frac{(\Sigma d)^2}{n(n-1)}} = \sqrt{\frac{55244}{8-1} - \frac{(664)^2}{8(8-1)}}$$

$= 4.342$

Applying test statistic

$$t = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{83-87}{4.342/\sqrt{8}} = -2.61$$

Since t_{cal} (= –2.61) < t (= – 1.895) at df = 7, reject H_0. Hence, the kilometer is less than advertiser.

10.67 H_0: σ_1^2 and σ_2^2 and H_1: $\sigma_1^2 > \sigma_2^2$.

Given $s_1^2 = 1.04$, $s_2^2 = 0.51$, $n_1 = n_2 = 25$. Applying the test statistic

$$f = s_1^2 / s_2^2 = 1.04/0.51 = 2.04$$

Since f_{cal} (= 2.04) > f_α (= 1.70) at $df_1 = df_2 = 24$, reject H_0. Hence variability of line 2 is less than that of line 1.

16.68 H_0: $\sigma_L^2 = \sigma_L^2$ and H_0: $\sigma_L^2 \neq \sigma_L^2$

Given $s_w^2 = 12$, $s_m^2 = 10$, $n_m = 10$, $n_w = 12$. Applying the test statistic

$$f = s_1^2 / s_2^2 = (12)^2/(10)^2 = 1.44$$

Since f_{cal} (= 1.44) < f_α (= 3.10), accept H_0. Hence there is no difference in the variations of the two populations.

People can be divided into three groups: those who make things happen, those who watch things happen, and those who wonder what happened.

—John W. Newbern

An approximate answer to the right problem is worth a good deal more than an exact answer to an approximate problem.

—John Tukey

Chapter 11

Chi-Square and Other Non-Parametric Tests

LEARNING OBJECTIVES

After studying this chapter, you should be able to

- recognize the advantage and disadvantages of non-parametric statistical tests
- learn how a non-parametric statistical test is conducted when
  - variables are measured on a nominal scale, and
  - measurements are of independent nature.
- test significant association between categorical variables.

11.1 INTRODUCTION

A variety of statistical tests are available for analysing a given set of data. An appropriate statistical test for analysing a given set of data is selected on the basis of

- scale of measurement of the data
- dependence/independence of the measurements
- number of populations being studied
- specific requirements such as sample size, shape of population distribution, and so on, for using a statistical test.

Generally, parametric and non-parametric statistical tests are distinguished on (i) the basis of the scaling of the data and (ii) the assumptions regarding the sampling distribution of sample statistic.

In Chapter 10, we discussed how z, t, and F test statistics are used for estimation and test of hypotheses about population parameters. The use of these tests

(i) require the level of measurement attained on the collected data in the form of an interval scale or ratio scale,

(ii) involve hypothesis testing of specified parameter values, and,

(iii) require assumptions about the population distribution, in particular, assumption of normality and whether standard deviation of sampling/population distribution is known or not.

If these assumptions are not justified then these tests would not yield accurate conculsions about population parameters. In such circumstances, it is necessary to use few other hypothesis testing procedures that do not require these conditions to be met. These procedures are referred to as *non-parametric tests*. **Non-parametric tests** (i) do not depend on the form of the underlying population distribution from which the samples were drawn, and (ii) use data that are of insufficient strength, i.e. data are categorical (nominally) scaled or ranks (ordinally) scaled.

Non-parametric (distribution-free) tests: The tests which can be used validly when the assumptions needed for parametric testing cannot be met.

A non-parametric procedure (or method), also called *distribution free test* satisfies at least one of the following criteria:

(i) The procedure does not take into consideration any population parameter such as μ, σ or p.
(ii) The procedure is applied only on categorical data that are non-numerical and frequency counts of categories for one or more variables.
(iii) The procedure does not depend on the form of the underlying population distribution, in paraticular, the requirement of normality.

11.2 ADVANTAGES AND LIMITATIONS OF NON-PARAMETRIC METHODS

Advantages Few advantages of using non-parametric methods are as under:

(i) Non-parametric methods can be used to analyse categorical (nominal scaling) data, rank (ordinal scaling) data and interval (ratio scaling) data
(ii) Non-parametric methods are generally easy to apply and quick to compute when sample size is small.
(iii) Non-parametric methods require few assumptions but very useful when the scale of measurement is weaker than required for parametric methods. Hence these methods are widely used and yield a more general, broad-base conclusions.
(iv) Non-parametric methods provide an approximate solution to an exact problem whereas parametric methods provide an exact solution to an approximate problem.
(v) Non-parametric methods provide solution to problems that do not require to make the assumption that a population is distributed normally or any specific shape.

Limitations: Few major limitations of non-parametric methods are as under.

(i) Non-parametric methods should not be used when all the assumption of the parametric methods can be met. However they are equally powerful when assumptions are met, when assumptions are not met these may be more powerful.
(ii) Non-parametric methods require more manual computational time when sample size gets larger.
(iii) Table values for non-parametric statistics are not as readily available as of parametric methods.
(iv) Non-parametric tests are usually not as widely used and not well known as parametric tests.

11.3 THE CHI-SQUARE DISTRIBUTION

The term non-parametric does not mean that the population distribution under study has no parameters. All populations have certain parameters which define their distribution. In this section we will discuss the **chi-square (χ^2) test** which belongs to non-parametric category of methods, to test a hypothesis. The symbol χ is the Greek letter 'chi'. The sampling distribution of χ^2 is called *χ^2-distribution*. Like other hypothesis testing procedures, the calculated value of χ^2-test statistic is compared with its critical (or table) value to know whether the null hypothesis is true. The decision of accepting a null hypothesis is based on how 'close' the sample results are to the expected results.

Chi-square test: A test for establishing the association between two categorical variables.

The probability density function of χ^2-distribution is given by

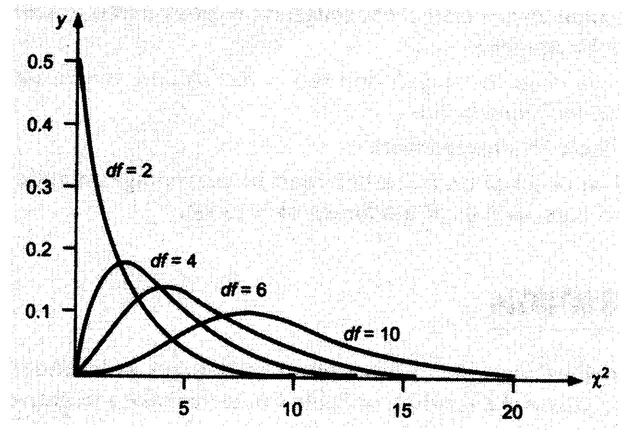
$$y = y_0\,(\chi^2)^{(\nu/2)-1}\,e^{-\chi^2/2} \qquad (11\text{-}1)$$

where ν = degrees of freedom ($df\,\nu$)

y_0 = a constant depending on degrees of freedom ν

e = a constant, 2.71828

Figure 11.1
χ^2-Distributions with different values of *df* parameter.



The χ^2-distribution is a continuous probability distribution extending from 0 to ∞ as shown in Fig. 11.1. Since χ^2 is the sum of squares, its value cannot be negative.

11.3.1 Properties of χ^2 Distribution

The following properties are useful while using χ^2-test statistic to analyse its sampling distribution:

1. The shape of the curve for various values of degrees of freedom is shown in Fig 11.1. For $v = 1$, the density function (11–1) reduces to
$$y = y_0 \; e^{-\chi^2/2}$$
which is the standard normal curve for positive values of the variate.
2. The sampling distribution of χ^2 is a family of curves which vary with degrees of freedom. When $v = 1$, the curve is tangential to the x-axis at the origin, that is, the curve attains its maximum value when
$$\frac{dy}{d\chi^2} = y_0 \, [(v-1) - \chi^2] \, \chi^{v-2} \, e^{-\chi^2/2} = 0$$
or $\qquad (v-1) - \chi^2 = 0 \quad \text{or} \quad \chi^2 = v - 1$
when $v > 1$, the curve fall slowly and $y \to 0$ as $\chi^2 \to \infty$. In other words, sampling distribution of χ^2 is skewed towards higher values, that is, positively skewed.
3. For degrees of freedom $v = 3$, the curve touches the y-axis at the origin, and for all other values of $v > 4$, the curve is tangential to χ^2 axis at the origin.
4. For degrees of freedom $v \geq 30$, the χ^2 curve approximates to the normal curve with mean v and standard deviation $\sqrt{2v}$. In such a case the distribution of $\sqrt{2\chi^2}$ provides a better approximation to normality than χ^2 with mean $\sqrt{2v-1}$ and standard deviation one. This characteristic helps to test the significance of the difference between observed and expected values of the variable.
5. Since density function of χ^2 does not contain any parameter of population, χ^2-test statistic is referred to as a non-parametric test. Thus χ^2-distribution does not depend upon the form of the parent population.
6. The mean and variance of χ^2-distribution are as follows:
Mean, $\mu(\chi^2) = v$ and Variance, $\sigma^2(\chi^2) = 2v$.

11.3.2 Conditions for the Applications of χ^2 Test

Before using χ^2 as a test statistic to test a hypothesis, the following conditions are necessary:

1. The experiment consists of n identical but independent trials. The outcome of each trial falls into one of k categories. The observed number of outcome in each category, written as $O_1, O_2, \ldots, O_n$, with $O_1 + O_2 + \ldots + O_n = 1$ are counted.
2. If there are only two cells, the expected frequency in each cell should be 5 or more. Because for observations less than 5, the value of χ^2 shall be over estimated, resulting in the rejection of the null hypothesis.

3. For more than two cells, if more than 20 per cent of the cells have expected frequencies less than 5, then χ^2 should not be applied.
4. Samples must be drawn randomly from the population of interest. All the individual observations in a sample should be independent.
5. The sample should contain at least 50 observations.
6. The data should be expressed in original units, rather than in percentage or ratio form. Such precaution helps in comparison of attributes of interest.

11.4 THE CHI-SQUARE TEST-STATISTIC

Like t and F distributions, a χ^2-distribution is also a function of its degrees of freedom. This distribution is skewed to the right and the random variable can never take a negative value. Theoretically, its range is from 0 to ∞ as shown in Fig. 11.2. Values of χ^2 that divide the curve with a proportion of the area equivalent to α (level of significance) in the right tail are given in the Appendix. The χ^2-test statistic is given by

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} \qquad (11\text{–}2)$$

where O = an observed frequency in a particular category
E = an expected frequency for a particular category

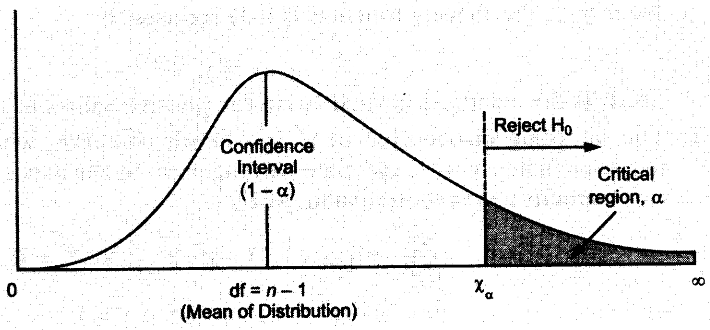


Figure 11.2
χ^2 Density Function

Decision Rule The calculated value of χ^2 is compared with its critical value at a particular level of significance and degrees of freedom. If $\chi^2_{cal} > \chi^2_{critical}$, then the null hypothesis is rejected in favour of the alternative hypothesis, and it is concluded that the difference between two sets of frequencies is significant.

The degrees of freedom for χ^2-test statistic depend on the test and certain other factors, which will be discussed later in this chapter.

Since the mean of χ^2-distribution is equal to the number of degrees of freedom, therefore skewness of this distribution is considerable when the number of degrees of freedom is small, but it reduces as the number of degrees of freedom increases as shown in Fig. 11.2.

11.4.1 Grouping of Small Frequencies

One or more observations with frequencies less than 5 may be grouped together to represent a single category before calculating the difference between observed and expected frequencies. For example, the figures given below are the theoretical (observed) and expected frequencies (based on Poisson distribution) having same mean value and equal number of total frequencies.

Observed frequencies :	305	365	210	80	28	9	3
Expected frequencies :	301	361	217	88	26	6	1

These 7 classes can be reduced to 6 by combining the last two frequencies in both the cases as follows:

Observed frequencies :	305	365	210	80	28	12
Expected frequencies :	301	361	217	88	26	7

Since the original 7 classes have been reduced to 6 by grouping, therefore the revised degrees of freedom are $df = 6 - 2 = 4$, due to two restraints.

11.5 APPLICATIONS OF χ^2 TEST

A few important applications of χ^2 test discussed in this chapter are as follows:

- Test of independence
- Test of goodness-of-fit
- Yate's correction for continuity
- Test for population variance
- Test for homogeneity

11.5.1 Contingency Table Analysis : Chi-Square Test of Independence

The χ^2 test of independence is used to analyse the frequencies of two qualitative variables or attributes with multiple categories to determine whether the two variables are independent. The chi-square test of independence can be used to analyse any level of measurement, but it is particularly useful in analysing nominal data. For example,

- Whether voters can be classified by gender is independent of the political affiliation
- Whether university students classified by gender are independent of courses of study
- Whether wage-earners classified by education level are independent of income
- Whether type of soft drink preferred by a consumer is independent of the consumer's age.
- Whether absenteeism is independent of job classification
- Whether an item manufactured is acceptable or not is independent of the shifts in which it was manufactured.

Contingency table: A cross-table for displaying the frequencies of all possible groups of two variables.

When observations are classified according to two qualitatives variables or attributes and arranged in a table, the display is called a **contingency table** as shown in Table 11.1. The test of independence uses the contingency table format and is also referred to as a *Contingency Table Analysis (or Test).*

Table 11.1: Contingency Table

Variable B	*Variable A*				*Total*
	A_1	A_2	...	A_c	
B_1	O_{11}	O_{12}	...	O_{1c}	R_1
B_2	O_{21}	O_{22}	...	O_{2c}	R_2
.	.				.
.	.				.
.	.				.
B_r	O_{r1}	O_{r2}	...	O_{rc}	R_r
Total	C_1	C_2	...	C_c	N

It may be noted that the variables A and B have been classified into mutually exclusive categories. The value O_{ij} is the observed frequency for the cell in row i and colum j. The row and column totals are the sums of the frequencies. The row and column totals are added up to get a grand total n, which represents the sample size.

The *expected frequency*, E_{ij}, corresponding to an observed frequency O_{ij} in row i and column j under the assumption of independence, is based on the multiplicative rule of probability. That is, if two events A_i and B_j are independent, then the probability of their joint occurrence is equal to the product of their individual probabilities. Thus the expected frequencies in each cell of the contingency table are calculated as follows:

$$E_{ij} = \frac{\text{Row } i \text{ total}}{\text{Sample size}} \times \frac{\text{Column } j \text{ total}}{\text{Sample size}} \times \text{Grand total}$$

$$= \frac{R_i}{N} \times \frac{C_j}{N} \times N = \frac{R_i \times C_j}{N} \qquad (11\text{–}3)$$

The analysis of a two-way contingency table helps to answer the question whether the two variables are unrelated or independent of each other. Consequently, *the null hypothesis for a chi-square test of independence is that the two variables are independent.* If null hypothesis H_0 is rejected, then two variables are not independent but are related. Hence, the χ^2-test statistic measures how much the observed frequencies differ from the expected frequencies when the variables are independent.

The Procedure The procedure to test the association between two independent variables where the sample data is presented in the form of a contingency table with r rows and c columns is summarized as follows:

Step 1: State the null and alternative hypotheses

H_0 : No relationship or association exists between two variables, that is, they are independent

H_1 : A relationship exists, that is, they are related

Step 2: Select a random sample and record the observed frequencies (O values) in each cell of the contingency table and calculate the row, column, and grand totals.

Step 3: Calculate the expected frequencies (E-values) for each cell:

$$E = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$$

Step 4: Compute the value of test-statistic

$$\chi^2 = \Sigma \frac{(O - E)^2}{E}$$

Step 5: Calculate the degrees of freedom. The degrees of freedom for the chi-square test of independence are given by the formula

$$df = (\text{Number of rows} - 1)\,(\text{Number of columns} - 1) = (r - 1)\,(c - 1)$$

Step 6: Using a level of significance α and df, find the critical (table) value of χ^2_α (see Appendix). This value of χ^2_α corresponds to an area in the right tail of the distribution.

Step 7: Compare the calculated and table values of χ^2. Decide whether the variables are independent or not, using the decision rule:

- Accept H_0 if χ^2_{cal} is less than its table value $\chi^2_{\alpha,\,(r-1)\,(c-1)}$
- Otherwise reject H_0

Example 11.1: Two hundred randomly selected adults were asked whether TV shows as a whole are primarily entertaining, educational, or a waste of time (only one answer could be chosen). The respondents were categorized by gender. Their responses are given in the following table:

	Opinion			
Gender	*Entertaining*	*Educational*	*Waste of time*	*Total*
Female	52	28	30	110
Male	28	12	50	90
Total	80	40	80	200

Is this evidence convincing that there is a relationship between gender and opinion in the population interest?

Solution: Let us assume the null hypothesis that the opinion of adults is independent of gender.

The contingency table is of size 2×3, the degrees of freedom would be (2–1) (3–1) = 2, that is, we will have to calculate only two expected frequencies and the other four can be automatically determined as shown below:

$$E_{11} = \frac{\text{Row 1 total} \times \text{Column 1 total}}{\text{Grand total}} = \frac{110 \times 80}{200} = 44$$

$$E_{12} = \frac{\text{Row 1 total} \times \text{Column 2 total}}{\text{Grand total}} = \frac{110 \times 40}{200} = 22$$

$$E_{13} = 110 - (44 + 22) = 44$$

$$E_{21} = 80 - E_{11} = 80 - 44 = 36$$

$$E_{22} = 40 - E_{12} = 40 - 22 = 18$$

$$E_{23} = 80 - E_{13} = 80 - 44 = 36$$

The contingency table of expected frequencies is as follows:

	Opinion			
Gender	*Entertaining*	*Educational*	*Waste of time*	*Total*
Male	44	22	44	110
Female	36	18	36	90
Total	80	40	80	200

Arranging the observed and expected frequencies in the following table to calculate the value of χ^2-test statistic:

Observed (O)	*Expected (E)*	*(O – E)*	$(O-E)^2$	$(O-E)^2/E$
52	44	8	64	1.454
28	22	6	36	1.636
30	44	14	196	4.455
28	36	– 8	64	1.777
12	18	– 6	36	2.000
50	36	14	196	5.444
				16.766

The critical (or table) value of $\chi^2 = 5.99$ at $\alpha = 0.05$ and $df = 2$. Since the calculated value of $\chi^2 = 16.766$ is more than its critical value, the null hypothesis is rejected. Hence, we conclude that the opinion of adults is not independent of gender.

Example 11.2: A company is interested in determining whether an association exists between the commuting time of their employees and the level of stress-related problems observed on the job. A study of 116 assembly-line workers reveals the following:

	Stress			
Commuting Time	*High*	*Moderate*	*Low*	*Total*
Under 20 min	9	5	18	32
20 – 50 min	17	8	28	53
over 50 min	18	6	7	31
Total	44	19	53	116

At $\alpha = 0.01$ level of significance, is there any evidence of a significant relationship between commuting time and stress?

Solution: Let us assume the null hypothesis that stress on the job is independent of commuting time.

The contingency table is of size 3×3, the degrees of freedom would be (3 – 1) (3 – 1) = 4, that is, we will have to calculate only four expected frequencies and the others can be calculated automatically as shown below:

$$E_{11} = \frac{32 \times 44}{116} = 12.14 \quad E_{12} = \frac{32 \times 19}{116} = 5.24 \quad E_{13} = 14.62$$

$$E_{21} = \frac{53 \times 44}{116} = 20.10 \quad E_{22} = \frac{53 \times 19}{116} = 8.68 \quad E_{23} = 24.22$$

$$E_{31} = \frac{31 \times 44}{116} = 11.75 \quad E_{32} = \frac{31 \times 19}{116} = 5.08 \quad E_{33} = 14.17$$

Arranging the observed and expected frequencies in the following table to calculate the value of χ^2-test statistic:

Observed (*O*)	*Expected* (*E*)	$O - E$	$(O - E)^2$	$(O - E)^2/E$
9	12.14	−3.14	9.85	0.811
5	5.24	−0.24	0.05	0.009
18	14.62	3.38	11.42	0.781
17	20.10	−3.10	9.61	0.478
8	8.68	−0.68	0.45	0.052
28	24.22	3.78	14.28	0.589
18	11.75	6.25	39.06	3.324
6	5.08	0.92	0.84	0.165
7	14.17	−7.17	51.40	3.627
				9.836

The critical value of $\chi^2 = 13.30$ at $\alpha = 0.01$ and $df = 4$. Since calculated value of $\chi^2 = 9.836$ is less than its critical value, the null hypothesis H_0 is accepted. Hence we conclude that stress on the job is independent of commuting time.

Example 11.3: A certain drug is claimed to be effective in curing colds. In an experiment on 500 persons with cold, half of them were given the drug and half of them were given sugar pills. The patients' reactions to the treatment are recorded in the following table:

Treatment	*Consequence*			*Total*
	Helped	*Reaction*	*No effect*	
Drug	150	30	70	250
Sugar pills	130	40	80	250
Total	280	70	150	500

On the basis of the data, can it be concluded that there is a significant difference in the effect of the drug and sugar pills?

[*Lucknow Univ., MBA, 1998, Delhi Univ., MBA,1999, 2002*]

Solution: Let us assume the null hypothesis that there is no significant difference in the effect of the drug and sugar pills.

The contingency table is of size 2 × 3, the degrees of freedom would be (2 − 1) (3 − 1) = 2, that is, we would have to calculate only two expected frequencies and others can be automatically determined as shown below:

$$E_{11} = \frac{250 \times 280}{500} = 140; \quad E_{12} = \frac{250 \times 70}{500} = 35$$

The contingency table of expected frequencies is as follows:

Treatment	*Consequence*			*Total*
	Helped	*Harmed*	*No Effect*	
Drug	140	35	75	250
Sugar pills	140	35	75	250
Total	280	70	150	500

Arranging the observed and expected frequencies in the following table to calculate the value of χ^2-test statistic.

Observed (O)	*Expected* (E)	$(O-E)$	$(O-E)^2$	$(O-E)^2/E$
150	140	10	100	0.714
130	140	– 10	100	0.714
30	35	– 5	25	0.714
40	35	5	25	0.714
70	75	– 5	25	0.333
80	75	5	25	0.333
				3.522

The critical value of $\chi^2 = 5.99$ at $\alpha = 0.05$ and $df = 2$. Since the calculated value of $\chi^2 = 3.522$ is less than its critical value, the null hypothesis is accepted. Hence we conclude that there is no significant difference in the effect of the drug and sugar pills.

Self-Practice Problems 11A

11.1 In an anti-malaria campaign in a certain area, quinine was administered to 812 persons out of a total population of 3248. The number of fever cases reported is shown below:

Treatment	*Fever*	*No Fever*	*Total*
Quinine	20	792	812
No quinine	220	2216	2436
Total	240	3008	3248

Discuss the usefulness of quinine in checking malaria.
[*MD Univ., MCom, 1998*]

11.2 Based on information from 1000 randomly selected fields about the tenancy status of the cultivation of these fields and use of fertilizers, collected in an agro-economic survey, the following classifications were noted:

	Owned	*Rented*	*Total*
Using fertilizers	416	184	600
Not using fertilizers	64	336	400
Total	480	520	1000

Would you conclude that owner cultivators are more inclined towards the use of fertilizers at $\alpha = 0.05$ level of significance? Carry out the chi-square test as per testing procedures. [*Osmania Univ., MCom, 1997*]

11.3 In an experiment on immunization of cattle from tuberculosis, the following results were obtained:

	Affected	*Not Affected*
Inoculated	12	26
Not inoculated	16	6

Calculate the χ^2 and discuss the effect of vaccine in controlling susceptibility to tuberculosis.
[*Rajasthan Univ., MCom, 1996*]

11.4 From the data given below about the treatment of 250 patients suffering from a disease, state whether the new treatment is superior to the conventional treatment.

Treatment	*No. of Patients*		
	Favourable	*Not Favourable*	*Total*
New	140	30	170
Conventional	60	20	80
Total	200	50	250

[*MD Univ., MCom, 1998*]

11.5 1000 students at college level are graded according to their IQ and their economic conditions. Use chi-square test to find out whether there is any association between economic conditions and the level of IQ.

Economic Conditions	*IQ Level*			
	High	*Medium*	*Low*	*Total*
Rich	160	300	140	600
Poor	140	100	160	400
Total	300	400	300	1000

[*Madurai Univ., MCom, 1996*]

11.6 200 digits are chosen at random from a set of tables. The frequencies of the digits are as follows:

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	22	20	21	15

Use the χ^2 test to assess the correctness of the hypothesis that the digits were distributed in equal numbers in the tables from which they were chosen.
[*HP Univ., MCom, 2000*]

11.7 In an experiment on pea-breeding, Mendel obtained the following frequencies of seeds: 315 round and yellow, 101 wrinkled and yellow, 108 round and green, 32 winkled and green. According to his theory of heredity the numbers should be in proportion 9 : 3 : 3 : 1. Is there any evidence to doubt the theory at $\alpha = 0.05$ level of significance?

[*Delhi Univ., MCom, 1996*]

11.8 Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling in different intelligence levels. The results are as follows:

Researcher	*Below Average*	*Average*	*Above Average*	*Genius*	*Total*
X	86	60	44	10	200
Y	40	33	25	2	100
Total	126	93	69	12	300

Would you say that the sampling techniques adopted by the two researchers are significantly different?

[*Delhi Univ., MBA, 1998*]

11.9 From the following data, find out whether there is any relationship between sex and preference of colour:

Colour	*Males*	*Females*	*Total*
Red	10	40	50
White	70	30	100
Green	30	20	50
Total	110	90	200

[*HP Univ., MBA., 1998; Madurai Univ., MCom, 1999*]

11.10 A manufacturer of TV sets was trying to find out what variables influenced the purchase of a TV set. Level of income was suggested as a possible variable influencing the purchase of TV sets. A sample of 500 households was selected and the information obtained is classified as shown below:

Income Group	*Have TV Set*	*Do not have TV Set*
Low income group	0	250
Middle income group	50	100
High income group	80	20

Is there evidence from the above data of a relation ownership of TV sets and level of income?

[*Delhi Univ., MBA, 2000*]

11.11 A marketing agency gives the following information about the age groups of the sample informants and their liking for a particular model of scooter which a company plans to introduce:

Choice	*Age Group of Informants*			
	Below 20	*20–39*	*40–59*	*Total*
Liked	125	420	60	605
Disliked	75	220	100	395
Total	200	640	160	1000

On the basis of above data, can it be concluded that the model appeal is independent of the age group of the informants? [*HP Univ., MBA, 1998*]

Hints and Answers

11.1 Let H_0 : Quinine is not effective in checking malaria.
$\chi^2_{cal} = 38.393$ is more than its critical value $\chi^2_{critical} = 3.84$ at a = 0.05 and $df = 1$, the null hypothesis is rejected.

11.2 Let H_0 : Ownership of fields and the use of fertilizers are independent attributes.
$\chi^2_{cal} = 273.504$ is more than its critical value $\chi^2_{critical} = 3.84$ at $\alpha = 0.05$ and $df = 1$, the null hypothesis is rejected.

11.3 Let H_0: Vaccine is not effective in controlling susceptibility to tuberculosis.
$\chi^2_{cal} = 7.796$ is more than its critical value $\chi^2_{critical} = 3.84$ at $\alpha = 0.05$ and $df = 1$, the null hypothesis is rejected.

11.4 Let H_0 : No significant difference between the new and conventional treatment.
$\chi^2_{cal} = 1.839$ is less than its critical value $\chi^2_{critical} = 3.84$ at $\alpha = 0.05$ and $df = 1$, the null hypothesis is accepted.

11.5 Let H_0: No association between economic conditions and the level of IQ.
$\chi^2_{cal} = 65.277$ is more than its critical value $\chi^2_{critical} = 5.99$ at $\alpha = 0.05$ and $df = 2$, the null hypothesis is rejected.

11.6 Let H_0 : Digits were distributed in equal numbers.
On the basis of H_0, E = 200/10 = 20 as the frequency of occurrence for 0, 1, 2, . . . digits.
$\chi^2_{cal} = 4.3$ is less than its critical value $\chi^2 = 16.22$ at $\alpha = 0.05$ and $df = 10 - 1 = 9$, the null hypothesis is accepted.

11.7 Let H_0 : No significant difference in the observed and expected frequencies.
Calculations of expected values are as follows:

$$E_1 = \frac{556 \times 9}{16} = 312.75,\ E_2 = \frac{556 \times 3}{16} = 104.25,$$

$E_3 = 104.25$ and $E_4 = 34.75$, respectively.

Category	*O*	*E*	$(O-E)^2$	$(O-E)^2/E$
Round and yellow	315	312.75	5.062	0.016
Wrinkled and yellow	101	104.25	10.562	0.101
Round and green	108	104.25	14.062	0.135
Wrinkled and green	32	34.75	7.562	0.218
				0.470

Since $\chi^2_{cal} = 0.470$ is less than its critical value $\chi^2 = 7.82$ at $\alpha = 0.05$ and $df = 4 - 1 = 3$, the null hypothesis is accepted.

11.8 Let H_0 : Sampling techniques adopted by two researchers are not significantly different.

χ^2_{cal} = 2.098 is less than its critical value $\chi^2_{critical}$ = 7.82 at α = 0.05 and df = 3, the null hypothesis is accepted.

11.9 Let H_0 : No relationship between gender and preference of colour.

χ^2_{cal} = 34.35 is more than its critical value $\chi^2_{critical}$ = 5.99 at α = 0.05 and df = 2, the null hypothesis is rejected.

11.10 Let H_0 : The ownership of TV sets is independent of the level of income.

χ^2_{cal} = 243.59 is more than its critical value $\chi^2_{critical}$ = 7.82 at α = 0.05 and df = 2, the null hypothesis is rejected.

11.11 Let H_0 : The model appeal is independent of the age group of the informants.

χ^2_{cal} = 42.788 is more than its critical value $\chi^2_{critical}$ = 5.99 at α = 0.05 and df = 2, the null hypothesis is rejected.

11.5.2 Chi-Square Test for Goodness-of-Fit

Goodness-of-fit: A statistical test conducted to determine how closely the observed frequencies fit those predicted by a hypothesized probability distribution for population.

On several occasions a decision-maker needs to understand whether an actual sample distribution matches or coincides with a known theoretical probability distribution such as binomial, Poisson, normal, and so on. *The χ^2 test for* **goodness-of-fit** *is a statistical test of how well given data support an assumption about the distribution of a population or random variable of interest. The test determines how well an assumed distribution fits the given data.* To apply this test, a particular theoretical distribution is first hypothesized a for given population and then the test is carried out to determine whether or not the sample data could have come from the population of interest with the hypothesized theoretical distribution. The observed frequencies or values come from the sample and the expected frequencies or values come from the theoretical hypothesized probability distribution. The goodness-of-fit test now focuses on the differences between the observed values and the expected values. Large differences between the two distributions throw doubt on the assumption that the hypothesized theoretical distribution is correct. On the other hand, small differences between the two distribution may be assumed to be resulting from sampling error.

The Procedure The general steps to conduct a goodness-of-fit test for any hypothesized population distribution are summarized as follows:

Step 1: State the null and alternative hypotheses

H_0 : No difference between the observed and expected sets of frequencies.

H_1 : There is a difference

Step 2: Select a random sample and record the observed frequencies (O values) for each category.

Step 3: Calculate expected frequencies (E values) in each category by multiplying the category probability by the sample size.

Step 4: Compute the value of test statistic

$$\chi^2 = \Sigma \frac{(O - E)^2}{E}$$

Step 5: Using a level of significance α and $df = n - 1$ provided that the number of expected frequencies are 5 or more for all categories, find the critical (table) value of χ^2 (See Appendix).

Step 6: Compare the calculated and table value of χ^2, and use the following decision rule:

- Accept H_0 if χ^2_{cal} is less than its critical value $\chi^2_{\alpha, n-1}$
- Otherwise reject H_0

Example 11.4: A Personnel Manager is interested in trying to determine whether absenteeism is greater on one day of the week than on another. His records for the past year show the following sample distribution:

Day of the week :	Monday	Tuesday	Wednesday	Thursday	Friday
No. of absentees :	66	56	54	48	75

Test whether the absence is uniformly distributed over the week.

[Madras Univ., MCom, 1996]

Solution: Let us assume the null hypothesis that the absence is uniformly distributed over the week.

The number of absentees during a week are 300 and if absenteeism is equally probable on all days, then we should expect 300/5 = 60 absentees on each day of the week. Now arranging the data as follows:

Category	*O*	*E*	*O – E*	$(O-E)^2$	$(O-E)^2/E$
Monday	66	60	6	36	0.60
Tuesday	57	60	– 3	9	0.15
Wednesday	54	60	– 6	36	0.60
Thursday	48	60	– 12	144	2.40
Friday	75	60	– 15	225	3.75
					7.50

The critical value of $\chi^2 = 9.49$ at $\alpha = 0.05$ and $df = 5 - 1 = 4$. Since calculated value $\chi_{cal} = 7.50$ is less than its critical value, the null hypothesis is accepted.

Example 11.5: A survey of 800 families with 4 children each revealed following distribution:

No. of boys	:	0	1	2	3	4
No. of girls	:	4	3	2	1	0
No. of families	:	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births are equally probable?

Solution: Let us assume the null hypothesis that male and female births are equally probable.

The probability of a male or female is $p = q = 1/2$. Since birth of male and female is mutually exclusive and exhaustive, the expected number of families having different combinations of boys and girls can be calculated using binomial probability distribution as follows:

$$P(x = r) = {}^4C_r\, p^r q^{4-r}\,;\ r = 0, 1, \ldots, 4$$
$$= {}^4C_r\,(1/2)^4 \text{ since } p = q = 1/2$$

The calculations for expected frequencies for each combination (category) of boy or girl are as shown below:

Category *x*	*P(x = r)*	*Expected Frequency, n P(x)*
0	${}^4C_0\,(1/2)^4 = 1/16$	$800\times(1/16) = 50$
1	${}^4C_1\,(1/2)^4 = 4/16$	$800\times(4/16) = 200$
2	${}^4C_2\,(1/2)^4 = 6/16$	$800\times(6/16) = 300$
3	${}^4C_3\,(1/2)^4 = 4/16$	$800\times(4/16) = 200$
4	${}^4C_4\,(1/2)^4 = 1/16$	$800\times(1/16) = 50$

To apply the χ^2-test, arrange the observed and expected frequencies in the table below:

Category	*O*	*E*	*O – E*	$(O-E)^2$	$(O-E)^2/E$
0	32	50	– 18	324	6.480
1	178	200	– 22	484	2.420
2	290	300	– 10	100	0.333
3	236	200	36	1296	6.480
4	64	50	14	196	3.920
					19.633

The critical value of χ^2 = 9.488 at α = 0.05 and df = 5 – 1 = 4. Since calculated value of χ^2 is greater than its table value, the hypothesis is rejected. Hence, we conclude that male and female births do not seem to be equally probable.

Example 11.6: The figures given below are (a) the theoretical frequencies of a distribution, and (b) the frequencies of the normal distribution having the same mean, standard deviation, and the total frequency as in (a):

(a)	1	5	20	28	42	22	15	5	2
(b)	1	6	18	25	40	25	18	6	1

Do you think that the normal distribution provides a good fit to the data?

Solution: Let us assume the null hypothesis that there is no difference between observed frequencies and expected frequencies obtained by normal distribution.

Since the observed and expected frequencies are less than 10 in the beginning and end of the serieses, we shall group these classes together as follows:

O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
1, 5 = 6	1, 6 = 7	–1	1	0.143
20	18	2	4	0.222
28	25	3	9	0.360
42	40	2	4	0.100
22	25	– 3	9	0.360
15	18	– 3	9	0.500
5, 2 = 7	6, 1 = 7	0	0	0.000
				1.685

The revised degrees of freedom are df = 9 – 1 – 4 = 4 is 4. The critical value χ^2_{critical} = 9.49 at α = 0.05 and df = 4. Since the calculated value of χ^2 = 1.685 is less than its critical value, the null hypothesis is accepted. Hence, we conclude that the normal distribution provides a good fit to the data.

Example 11.7: A book has 700 pages. The number of pages with various numbers of misprints is recorded below:

No. of misprints	:	0	1	2	3	4	5
No. of pages with misprints	:	616	70	10	2	1	1

Can a Poisson distribution be fitted to this data? *[Delhi Univ., MCom, 1999]*

Solution: Let us assume the null hypothesis that the data is fitted with Poisson distribution.

The number of pages in the book are 700, whereas the maximum possible number of misprints are only 5. Thus we may apply Poisson probability distribution to calculate the expected number of misprints in each page of the book as follows:

Mistakes (x)	Number of Pages (f)	fx
0	616	0
1	70	70
2	10	20
3	2	6
4	1	4
5	1	5
	n = 700	105

$$\lambda \text{ or } m = \frac{\Sigma fx}{n} = \frac{105}{700} = 0.15$$

The calculations for expected frequencies for misprints from 0 to 5 are shown below:

$$P(x = 0) = e^{-\lambda} = e^{-0.15} = 0.8607 \text{ (from table)}$$
$$nP(x = 0) = 700 \times 0.8607 = 602.5$$
$$nP(x = 1) = nP(x = 0)\ \lambda = 602.5 \times 0.15 = 90.38$$
$$nP(x = 2) = nP(x = 1)\ \frac{\lambda}{2} = 90.38 \times \frac{0.15}{2} = 6.78$$
$$nP(x = 3) = nP(x = 2)\ \frac{\lambda}{3} = 6.78 \times \frac{0.15}{3} = 0.34$$
$$nP(x = 4) = nP(x = 3)\ \frac{\lambda}{4} = 0.34 \times \frac{0.15}{4} = 0.013$$
$$nP(x = 5) = nP(x = 4)\ \frac{\lambda}{5} = 0.013 \times \frac{0.15}{5} = 0$$

To apply the χ^2-test, arrange the observed and expected frequencies in the table below:

Mistake	O	E	O − E	$(O-E)^2$	$(O-E)^2/E$
0	616	602.5	13.50	182.25	0.302
1	70	90.38	20.38	415.34	4.595
2	10	6.78	3.22	10.37	1.529
3	2 ⎤	0.34 ⎤	3.65	13.32	37.733
4	1 ⎥ = 4	0.013 ⎥ = 0.353			
5	1 ⎦	0 ⎦	—	—	—
					44.159

The table value of $\chi^2 = 5.99$ at $\alpha = 0.05$ and $df = 6 - 1 - 3 = 2$. Since calculated value of $\chi^2 = 37.422$ is greater than the its table value, the null hypothesis is rejected

11.5.3 Yate's Correction for Continuity

The distribution of χ^2-test statistic is continuous but the data under test is categorical which is discrete. It obviously causes errors, but it is not serious unless we have one degree of freedom, as in a 2 × 2 contingency table. To remove the probability of such errors to occur due to the effect of discrete data, we apply **Yate's correction** for continuity.

Yate's correction: A continuity correction made when calculating the χ^2-test statistic for a 2 × 2 contingency table.

The correction factor suggested by Yate in case of a 2 × 2 contingency table is as follows:

(a) Decrease by half those cell frequencies which are greater than expected frequencies and increase by half those which are less than expected.

This correction does not affect the row and column totals. For example, in a 2 × 2 contingency table where frequencies are arranged as:

Attributes	A	Not A	Total
B	a	b	$a + b$
Not B	c	d	$c + d$
Total	$a + c$	$b + d$	$a + b + c + d = N$

The value of χ^2 calculated from independent frequencies is given by

$$\chi^2 = \frac{(a+b+c+d)(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

This value of χ^2 is corrected as:

$$\chi^2_{\text{corrected}} = \frac{n\left(ad - bc - \frac{1}{2}n\right)^2}{(a+b)(c+d)(a+c)(b+d)}\ ; \quad ad - bc > 0$$

and
$$\chi^2_{\text{corrected}} = \frac{n\left(bc - ad - \frac{1}{2}n\right)^2}{(a+b)(c+d)(a+c)(b+d)}\ ; \quad ad - bc < 0$$

(b) An alternative formula for calculating the χ^2 test statistic is as follows:

$$\chi^2 = \Sigma \frac{\{|O - E| - 0.5\}^2}{E}$$

Example 11.8: Of the 1000 workers in a factory exposed to an epidemic, 700 in all were attacked, 400 had been inoculated, and of these 200 were attacked. On the basis of this information, can it be believed that inoculation and attack are independent?

[*HP Univ., MBA, 1998*]

Solution: Let us assume the null hypothesis that there is no association between inoculation and attack that is, inoculation and attack are independent.

The given information can be arranged in a 2 × 2 contingency table as follows:

Attributes	*Attacked*	*Not Attacked*	*Total*
Innoculated	200	200	400
Not innoculated	500	100	600
Total	700	300	1000

Using the result of Section 11.4.3, we have a = 200, b = 200, c = 500, d = 100, and n = 1000. Thus

$$\chi^2 = \frac{(a+b+c+d)(ab-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

$$= \frac{1000\,(200 \times 100 - 200 \times 500)^2}{(200+200)(500+100)(200+500)(200+100)}$$

$$= \frac{1000 \times (80{,}000)^2}{400 \times 600 \times 700 \times 300} = \frac{1000 \times 64}{504} = 126.984$$

The critical value of χ^2 = 3.84 at α = 0.05 and df = (2 – 1) (2 – 1) = 1. Since calculated value χ^2_{cal} = 126.984 is more than its critical value, the null hypothesis rejected. Hence, we conclude that innoculation and attack are not independent.

Example 11.9: The following information was obtained in a sample of 40 small general shops:

Owner	*Shops in* *Urban Areas*	*Rural Areas*	*Total*
Men	17	18	35
Women	2	12	15
Total	20	30	50

Can it be said that there are relatively more women owners of small general shops in rural than in urban areas?

Solution: Let us assume the null hypothesis that there are an equal number of men and women owners of small shops in both rural and urban areas.

Since one of the frequencies is small, we apply Yate's correction formula to calculate χ^2 as follows: Here a = 17, b = 18, c = 3, d = 12, and n = 50

$$\chi^2_{\text{corrected}} = \frac{n\left(ad - bc - \frac{1}{2}n\right)^2}{(a+b)(c+d)(a+c)(b+d)}$$

$$= \frac{50\left(17 \times 12 - 18 \times 3 - \frac{1}{2} \times 50\right)^2}{35 \times 15 \times 20 \times 30}$$

$$= \frac{50\,(204 - 54 - 25)^2}{35 \times 15 \times 20 \times 30} = \frac{50 \times 125 \times 125}{35 \times 15 \times 20 \times 30} = 2.480$$

The critical value of $\chi^2 = 3.841$ at $\alpha = 0.05$ and $df = (2-1)(2-1) = 1$. Since calculated value $\chi^2 = 2.480$ is less than its critical value $\chi^2 = 3.841$, the null hypothesis is accepted. Hence we conclude that shops owned by men and women in both areas are equal in number.

Self-Practice Problems 11B

11.12 A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured a third class, 90 were placed in second class, and 20 got a first class. Are these figures commensurate with the general examination result which is in the ratio of 4 : 3 : 2 : 1 for the various categories respectively. *[Delhi Univ., MCom, 1999]*

11.13 A set of 5 coins is tossed 3200 times, and the number of heads appearing each time is noted. The results are given below:

No. of heads :	0	1	2	3	4	5
Frequency :	80	570	1100	900	50	50

Test the hypothesis that the coins are unbiased.

[Annamalai Univ., MCom, 1998; MD Univ., MCom, 1998]

11.14 The demand for a particular spare part in a factory was found to vary from day to day. In a sample study the following information was obtained:

Day :	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
No. of parts demanded :	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week.

[Delhi Univ., MBA, 2002]

11.15 The number of scooter accidents per week in a certain town were as follows:

12 8 20 2 14 10 15 6 9 4

Are these frequencies in agreement with the belief that accident conditions were the same during this 10-week period?

[Delhi Univ., MBA, 2000; HP Univ., MA Econ, 1997]

11.16 The divisional manager of a chain of retail stores believes the average number of customers entering each of the five stores in his division weekly is the same. In a given week, a manager reports the following number of customers in his stores : 3000, 2960, 3100, 2780, 3160. Test the divisional manager's belief at the 10 per cent level of significance *[Delhi Univ., MBA, 2001]*

11.17 Figures given below are (a) the theoretical frequencies of a distribution and (b) the frequencies of the Poisson distribution having the same mean and total frequency as in (a).

(a)	305	365	210	80	28	9	3
(b)	301	361	217	88	26	6	1

Apply the χ^2 test for goodness-of-fit.

Hints and Answers

11.12 Let H_0 : No difference in observed and expected results.

Category	*O*	*E*	$(O-E)^2$	$(O-E)^2/E$
Failed	220	500(4/10) = 200	400	2.000
3rd class	170	500(3/10) = 150	400	2.667
2nd class	90	500(2/10) = 100	100	1.000
1st class	20	500(1/10) = 50	900	18.000
				23.667

Since $\chi^2_{cal} = 23.667$ is more than its critical value $\chi^2 = 7.81$ for $df = 4-1 = 3$ and $\alpha = 0.05$, the null hypothesis is rejected.

11.13 Let H_0 : Coins are unbiased that is, $p = q = 1/2$.

Apply binomial probability distribution to get the expected number of heads as follows:

Expected number of heads $= n\,{}^nC_r\,p^r q^{n-r}$

$$= 3200\ {}^5C_0\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{5-r} = 3200\ {}^5C_0\left(\frac{1}{2}\right)^5$$

O	*E*	$(O-E)^2$	$(O-E)^2/E$
80	100	400	4.00
570	500	4900	9.80
1100	1000	10,000	10.00
900	1000	10,000	10.00
500	500	0	0.00
50	100	2500	25.00
			58.80

Since $\chi^2_{cal} = 58.80$ is more than its critical value $\chi^2 = 11.07$ for $df = 6-1 = 5$ and $\alpha = 0.05$, the null hypothesis is rejected.

11.14 Let H_0 : Number of parts demanded does not depend on the day of the week.

Expected number of parts demanded = 6720/6 = 1120 when all days are considered same.

Days	*O*	*E*	$(O-E)^2$	$(O-E)^2/E$
Monday	1124	1120	16	0.014
Tuesday	1125	1120	25	0.022
Wednesday	1110	1120	100	0.089
Thursday	1120	1120	0	0.000
Friday	1126	1120	36	0.032
Saturday	1115	1120	25	0.022
				0.179

Since $\chi^2_{cal} = 0.179$ is less than its critical value $\chi^2 = 11.07$ for $df = 6 - 1 = 5$ and $\alpha = 0.05$, the null hypothesis is accepted.

11.15 Let H_0 : Accident conditions were same during the period.

Expected number of accidents per week = (10 + 8 + 20 + . . . + 4)/10 = 10

O	*E*	$(O-E)^2$	$(O-E)^2/E$
12	10	4	0.40
8	10	4	0.40
20	10	100	10.00
2	10	64	6.40
14	10	16	1.60
10	10	0	0.00
15	10	25	2.50
6	10	16	1.60
9	10	1	0.10
4	10	36	3.60
			26.60

Since $\chi^2_{cal} = 26.60$ is more than its critical value $\chi^2 = 16.819$ for $df = 10 - 1 = 9$ and $\alpha = 0.05$, the null hypothesis is rejected.

11.16 Let H_0 : No significant difference in the number of customers entering each of the five stores.

Expected frequency of customers entering each store is 15,000/5 = 3000.

O	*E*	$(O-E)^2$	$(O-E)^2/E$
3000	3000	0	0
2960	3000	1600	0.533
3100	3000	10,000	3.333
2780	3000	48,400	16.133
3160	3000	25,600	8.533
			28.532

Since $\chi^2_{cal} = 28.532$ is more than its critical value $\chi^2 = 13.277$ for $df = 5 - 1 = 4$ at $\alpha = 0.05$, the null hypothesis is rejected.

11.17 Let H_0 : The Poisson distribution is a good fit to the given data

$$O: \; 305 \quad 365 \quad 210 \quad 80 \quad 28 \quad \underbrace{9 \quad 3}_{12}$$

$$E: \; 301 \quad 361 \quad 217 \quad 88 \quad 26 \quad \underbrace{6 \quad 1}_{7}$$

$\chi^2 = \Sigma(O - E)^2/E = 4.8$; $df = 7 - 1 - 2 = 4$; $\alpha = 0.05$;

$\chi^2_{cal} = 9.49 > \chi^2_{cal}$, the H_0 is accepted.

11.5.4 χ^2 Test for Population Variance

The assumption underlying the χ^2-test is that the population from which the samples are drawn is normally distributed. Let the variance of normal population be σ^2. The null hypothesis is setup as: $H_0 : \sigma^2 = \sigma_0^2$, where σ_0^2 is hypothesized value of σ^2.

If a sample of size n is drawn form this normal population, then variance of sampling distribution of mean $\bar{x}$ is given by $s^2 = \Sigma(x - \bar{x})^2/(n - 1)$. Consequently the value of χ^2-test statistic is determined as

$$\chi^2 = \frac{1}{\sigma^2} \Sigma (x - \bar{x})^2 = \frac{(n-1)s^2}{\sigma^2}$$

with $df = n - 1$ degrees of freedom.

Decision rule • If $\chi^2_{cal} < \chi^2_{\alpha/2}$, then accept H_0

• Otherwise reject H_0

Confidence Interval for Variance We can define 95 per cent, 99 per cent, and so on. Confidence limits and intervals for χ^2 test statistic using table values of χ^2. Same as t-distribution. Such limits of confidence helps to estimate population standard deviation in forms of sample standard deviation s with $(1 - \alpha)$ per cent confidence as follows:

or $$\frac{(n-1)}{\chi^2_{df,U}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{df,L}}$$

or $$\sqrt{\frac{(n-1)s^2}{\chi^2_{df,U}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{df,L}}}$$

or $$\frac{s\sqrt{n-1}}{x_{df,\,U}} \le \sigma \le \frac{s\sqrt{n-1}}{x_{df,\,L}}$$

where subscripts U and L stands for upper and lower tails proportion of area under χ^2-curve. For example for a 95 per cent confidence interval the lowest 2.5 per cent and highest 25 per cent of χ^2 distribution curve is excluded leaving the middle 95 per cent area.

The χ^2 for upper 2.5 per cent (= 0.025) area is obtained directly for standard χ^2 table. To obtain value of χ^2 for lower 2.5 per cent (= 0.025) area, look under the 0.975 column of χ^2 table for given *df*, because 1 – 0.975 = 0.025.

In case χ^2 table values are not available for larger *df* (≥ 30) values, then the values of $\chi^2_{df,v}$ for an appropriate confidence interval based on normal approximation can be obtained follows:

$$\chi^2_{df,U} = \mu + z\sigma\,(\chi^2) \qquad \text{and} \qquad \chi^2_{df,L} = \mu - z\sigma\,(\chi^2)$$

where, mean $\mu\,(\chi^2) = df$ and variance $\sigma^2\,(\chi^2) = 2df$

and $$\sigma\,(\chi^2) = \sqrt{v\,(\chi^2)}$$

Example 11.10: A random sample of size 25 from a population gives the sample standard deviation of 8.5. Test the hypothesis that the population standard deviation is 10.

Solution: Let us assume the null hypothesis that the population standard deviation $\sigma = 10$.

Given that $n = 25$; $s = 8.5$. Applying the χ^2 test statistic, we have

$$\chi^2 = \frac{(n-1)\,s^2}{\sigma^2} = \frac{(25-1)\,(8.5)^2}{(10)^2} = \frac{24\,(72.25)}{100} = 17.34$$

The critical value of $\chi^2 = 36.415$ at $df = 25 - 1 = 24$ and $\alpha = 0.05$. Since calculated of χ^2 is less than its critical value, the null hypothesis is accepted. Hence we conclude that the population variance is 10.

Example 11.11: A as a sample of 8 units from a normal population gives an unbiased estimate of population variance as 4.4. Find the 95 per cent confidence limits for population standard deviation σ.

Solution: Given that $\alpha = 5$ per cent, $n = 8$, $s^2 = 4.4$, $df = 8 - 1 = 7$. The confidence limits for σ^2 are as follows:

$$\frac{(n-1)\,s^2}{\chi^2_{df,\,U}} < \sigma^2 < \frac{(n-1)\,s^2}{\chi^2_{df,\,L}}$$

$$\frac{(8-1)4.4}{16.01} < \sigma^2 < \frac{(8-1)4.4}{1.69} \quad \text{or } 1.923 \le \sigma^2 \le 18.224$$

Consequently $$1.386 \le \sigma \le 4.269$$

Example 11.12: The standard deviation of lifetime of a sample of electric light bulbs is 100 hours. Find the 95 per cent confidence limits for the population standard deviation for such electric bulbs.

Solution: Given $\alpha = 5$ per cent, $n = 200$, $s = 100$, $df = n - 1 = 199$. The approximate 95 per cent confidence interval based on normal approximation requires that χ^2 values be approximated, where

$$\mu(\chi^2) = df = 199,\ \sigma^2\,(\chi^2) = 2df = 398, \text{ and } \sigma\,(\chi^2) = 19.949$$

Thus the approximate χ^2 values are

$$\chi^2_{df,\,U=0.025} = \mu\,(\chi^2) + z\sigma\,(\chi^2) = 199 + 1.96\,(19.949) = 238.10$$

$$\chi^2_{df,\,L=0.025} = \mu\,(\chi^2) - z\sigma\,(\chi^2) = 199 - 1.96\,(19.949) = 159.90$$

The 95 per cent confidence interval for the population standard deviation based on normal approximation of χ^2 values is given by

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{df,\,U}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{df,\,L}}}$$

$$\frac{100\sqrt{199}}{15.430} \le \sigma \le \frac{100\sqrt{199}}{12.645} = 91.42 \le \sigma \le 111.55$$

Hence we can be 95 per cent confident that the population standard deviation will be between 91.42 hours and 111.55 hours.

11.5.5 Coefficient of Contingency

If a null hypothesis of independence is rejected at a certain level of significance, then it implies dependence of attributes on each other. In such a case we need to determine the measure of dependence in terms of association or relationship. The measure of the degree of relationship (association) of attributes in a contingency table is given by

$$C = \sqrt{\frac{\chi^2}{\chi^2 + n}} = \sqrt{\frac{s-n}{s}}; \quad s = \Sigma\,(O/E)^2$$

This value of C is called the *coefficient of contingency*. A large value of C represents a greater degree of dependence or association between two attributes.

The maximum value which the coefficient of contingency C can take is $\sqrt{(k-1)/k}$, where k represents the number of rows and columns in the contingency table, such that $C = r = k$. When there is perfect dependence and $C = r = k$, the non-zero observed cell frequencies will occur diagonally and the calculated value of χ^2 will be as large as the sample size.

Example 11.13: 1000 students at college level were graded according to their IQ level and the economic condition of their parents.

Economic Condition	*IQ Level*		
	High	*Low*	*Total*
Rich	460	140	600
Poor	240	160	400
Total	700	300	1000

Use the coefficient of contingency to determine the amount of association between economic condition and IQ level.

Solution: Calculations for expected frequencies are as shown below:

O	*E*	*O – E*	*(O – E)*^{2}	*(O – E)*^{2}*/E*
460	420	40	1600	3.810
240	280	– 40	1600	5.714
140	180	– 40	1600	8.889
160	120	40	1600	13.333
				31.746

The coefficient of contingency, $C = \sqrt{\frac{\chi^2}{\chi^2 + n}} = \sqrt{\frac{31.746}{31.746 + 1000}} = 0.175$ implies the level of association between two attributes, IQ level and economic condition.

The maximum value of C for a 2 × 2 contingency table is

$$C_{max} = \sqrt{\frac{k-1}{k}} = \sqrt{\frac{2-1}{2}} = \sqrt{\frac{1}{2}} = 0.7071$$

which implies perfect dependence between the IQ level and economic condition of students.

11.5.6 Chi-Square Test of Homogeneity

The test of homogeneity is useful in a case when we intend to verify whether several populations are homogeneous with respect to some characteristic of interest. For example, we may like to know that the milk supplied by various companies has a particular ingredient in common or not. Hence, the test of homogeneity is useful in testing a null hypothesis that several populations are homogeneous with respect to a characteristic.

This test is different from the test of independence on account of the following reasons:

(i) Instead of knowing whether two attributes are independent or not, we may like to know whether different samples come from the same population.

(ii) Instead of taking only one sample, for this test two or more independent samples are drawn from each population.

(iii) When the characteristics to be compared consist of two categories, this test is similar to the test of hypothesis of difference between two population means or proportions.

To apply this test, first a random sample is drawn from each population, and then in each sample the proportion falling into each category or strata is determined. The sample data so obtained is arranged in a contingency table. The procedure for testing of hypothesis is same as discussed for test of independence.

Example 11.14: A movie producer is bringing out a new movie. In order to develop an advertising strategy, he wants to determine whether the movie will appeal most to a particular age group or whether it will appeal equally to all age groups. The producer takes a random sample from persons attending the preview of the new movie, and obtains the following results:

Opinion	*Age Groups*				
	Under 20	*20-39*	*40-59*	*60 and over*	*Total*
Liked the movie	146	78	48	28	300
Disliked the movie	54	22	42	22	140
Indifferent	20	10	10	20	60
Total	220	110	100	70	500

What inference will you draw from this data?

Solution: Let us assume the null hypothesis that the opinion of all age groups is same about the new movie.

The calculations for expected frequencies are as follow and displayed in the table below:

$E_{11} = \frac{300 \times 220}{500} = 132, \quad E_{12} = \frac{300 \times 110}{500} = 66 \quad E_{13} = \frac{300 \times 100}{500} = 60$

$E_{14} = 300 - (132 + 66 + 60)$

$E_{21} = \frac{140 \times 220}{500} = 61.6, \quad E_{22} = \frac{140 \times 110}{500} = 30.8 \quad E_{23} = \frac{140 \times 100}{500} = 28$

$E_{24} = 140 - (61.6 + 30.8 + 28) = 42$

$E_{31} = 220 - (132 + 61.6) = 26.4 \quad E_{32} = 110 - (66 + 30.8) = 13.2$

$E_{33} = 100 - (60 + 28) = 12 \quad E_{34} = 70 - (42 + 19.6) = 8.4 = 19.6$

O	*E*	*O – E*	$(O-E)^2$	$(O-E)^2/E$
146	132.0	14.0	196.00	1.485
54	61.6	– 7.6	57.76	0.938
20	26.4	– 6.4	40.96	1.552
78	66.0	12.0	144.00	2.182
22	30.8	– 8.8	77.44	2.514
10	13.2	– 3.2	10.24	0.776
48	60.0	– 12.0	144.00	2.400
42	28.0	14.0	196.00	7.000
10	12.0	– 2.0	4.00	0.333
28	42.0	– 14.0	196.00	4.667
22	19.6	2.4	5.76	0.294
20	8.4	11.6	134.56	16.019
				40.16